Linear Algebra

Problem Set 1 **2009**

Due Wednesday, 11 March 2009 at 10:00 AM in EE207. Be sure to print your name and student ID on your homework.

1. Section 2.1, Problem 18
   Find the matrix P that multiplies (x, y, z) to give (y, z, x). Find the matrix Q that multiplies (y, z, x) to bring back (x, y, z).
2. Section 2.1, Problem 20
   What 3 by 3 matrix E multiplies (x, y, z) to give $(x, y, z+x)$? What matrix E^{-1} multiplies (x, y, z) to give $(x, y, z-x)$? If you multiply (3, 4, 5) by E and then multiply E^{-1}, the two results are (_____) and (_____).
3. Section 2.3, Problem 3
   Which three matrices E_{21}, E_{31}, E_{32} put A into triangular form U?

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix} \quad \text{and} \quad E_{32}E_{31}E_{21}A = U.$$

4. Let $\begin{bmatrix} a & 0 & b & 2 \\ a & a & 4 & 4 \\ 0 & a & 2 & b \end{bmatrix}$ be the augmented matrix for a linear system. For what values of a and b does the system have (a) a unique solution, (b) no solution, and (c) infinite many solutions?
5. Section 2.3, Problem 30
   Find the triangular matrix E that reduces "*Pascal's matrix*" to a smaller Pascal:

$$E\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix}.$$

6. Section 2.4, Problem 7 (Think of the 4 ways to multiply matrices.)
   True or false. Give a specific example when false:
   (a) If columns 1 and 3 of B are the same, so are columns 1 and 3 of AB.
   (b) If rows 1 and 3 of B are the same, so are rows 1 and 3 of AB.
   (c) If rows 1 and 3 of A are the same, so are rows 1 and 3 of ABC.
   (d) $(AB)^2 = A^2B^2$.
7. Section 2.4, Problem 11

(3 by 3 matrices) Choose the only B so that for every matrix A

(a) $BA = 4A$

(b) $BA = 4B$

(c) BA has rows 1 and 3 of A reversed and row 2 unchanged.

(d) All rows of BA are the same as row 1 of A.

8. Section 2.4, Problem 16 (Show every step of your proof.)

   To prove that $(AB)C = A(BC)$, use the column vectors $\mathbf{b}_1,\ldots,\mathbf{b}_n$ of B. First suppose that C has only one column $\mathbf{c}$ with entries $c_1,\ldots,c_n$:

   AB has columns $A\mathbf{b}_1,\ldots,A\mathbf{b}_n$ and $B\mathbf{c}$ has one column $c_1\mathbf{b}_1+\ldots+c_n\mathbf{b}_n$.

   Then $(AB)\mathbf{c} = c_1A\mathbf{b}_1+\ldots+c_nA\mathbf{b}_n$ equals $A(c_1\mathbf{b}_1+\ldots+c_n\mathbf{b}_n) = A(B\mathbf{c})$. *Linearity* gives equality of those two sums, and $(AB)\mathbf{c} = A(B\mathbf{c})$. The same is true for all other ____ of C. Therefore $(AB)C = A(BC)$.

9. Without using Gaussian elimination, solve X by inspection. Identify the size of X first.

   (a) $$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 3 & 0 \\ 1 & 4 & 1 \end{bmatrix} X = \begin{bmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 4 & 4 & 1 \\ 6 & 5 & 1 \end{bmatrix}$$

   (b) $$X \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 3 & 3 \\ 2 & 3 & 4 & 5 \\ 2 & 2 & 2 & 2 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

10. Section 2.4, Problem 34

    Suppose you are given three equations as below:

    $$A\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad A\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad A\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

    Solve $A\mathbf{x} = \mathbf{b}$ when $\mathbf{b} = (3, 5, 8)$. Challenge problem: What is A?