Linear Algebra

Problem Set 2 **2009**

Due Wednesday, 18 March 2009 at 10:00 AM in EE207. Be sure to print your name and student ID on your homework.

1. Section 2.5, Problem 7
   If A has row 1 + row 2 = row 3, show that A is not invertible:
   (a) Explain why $A\mathbf{x}=(1,0,0)$ cannot have a solution.
   (b) Which right sides (b_1, b_2, b_3) might allow a solution to $A\mathbf{x}=\mathbf{b}$?
   (c) What happen to row 3 in elimination?
2. Section 2.5, Problem 10
   Find the inverses (in any legal way) of
   $$A=\begin{bmatrix}0&0&0&2\\0&0&3&0\\0&4&0&0\\5&0&0&0\end{bmatrix} \quad \text{and} \quad B=\begin{bmatrix}3&2&0&0\\4&3&0&0\\0&0&6&5\\0&0&7&6\end{bmatrix}.$$
   (Actually, you don't need to use Gauss-Jordan method. Find the inverses by inspection.)
3. Section 2.5, Problem 32
   The matrix has a remarkable inverse. Find A^{-1} by elimination on $\begin{bmatrix}A & I\end{bmatrix}$.
   Extend to a 5 by 5 "alternating matrix" and guess its inverse; then multiply to confirm.
   $$A=\begin{bmatrix}1&-1&1&-1\\0&1&-1&1\\0&0&1&-1\\0&0&0&1\end{bmatrix}.$$
4. Suppose that A is n by n, U is n by m, and V is m by n. By direct multiplication, prove that if $A=I_n-UV$, then
   $$A^{-1}=I_n+U(I_m-VU)^{-1}V.$$
5. Suppose the matrix $\begin{bmatrix}1&2&3&1&b\\2&5&3&a&0\\1&0&8&6&c\end{bmatrix}$ can be transformed to

$\begin{bmatrix} 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & d & -1 \\ 0 & 0 & 1 & 1 & e \end{bmatrix}$ with a series of row operations. Find *a*, *b*, *c*, *d*, and *e*.

6. Suppose $B=(I+A)^{-1}(I-A)$ and $A=\begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 3 & 0 & 0 \\ 0 & -4 & 5 & 0 \\ 0 & 0 & -6 & 7 \end{bmatrix}$. Find $(I+B)^{-1}$.

   (Please don't do a lot of calculations! Try removing $(I+A)^{-1}$ from the equation first.)

7. Section 2.6, Problem 13

   Compute *L* and *U* for the symmetric matrix

$$A=\begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}.$$

   Find four conditions on *a*, *b*, *c*, *d* to get $A=LU$ with four pivots.

8. Section 2.6, Problem 18

   If $A = LDU$ and also $A = L_1D_1U_1$ with all factors invertible, then $L=L_1$ and $D=D_1$ and $U=U_1$. *The factors are unique*.

   (a) Derive the equation $L_1^{-1}LD = D_1U_1U^{-1}$. Are the two sides triangular or diagonal?

   (b) Deduce $L=L_1$ and $U=U_1$ (they all have diagonal 1's). Then $D=D_1$.

9. Section 2.7, Problem 18

   (a) How many entries of *A* can be chosen independently, if $A=A^T$ is 5 by 5?

   (b) How do *L* and *D* (still 5 by 5) give the same number of choices?

   (c) How many entries can be chosen if *A* is *skew-symmetric*? ($A^T=-A$).

10. Solve *X* given that

$$X\begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 0 \\ 3 & 1 & -1 \end{bmatrix}=\begin{bmatrix} 1 & 2 & 0 \\ -3 & 1 & 5 \end{bmatrix}.$$