Linear Algebra

Problem Set 4 **2009**

Due Monday, 6 April 2009 at 12:00 AM in EE207. Be sure to print your name and student ID on your homework.

1. Section 3.4, Problem 1

   Execute the six steps of Worked Example 3.4 A to describe the column space and nullspace of A and the complete solution to $A\mathbf{x} = \mathbf{b}$:

$$A = \begin{bmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}.$$

2. Section 3.4, Problem 10

   Construct a 2 by 3 system $A\mathbf{x} = \mathbf{b}$ with particular solution $\mathbf{x}_p = (2, 4, 0)$ and homogeneous solution $\mathbf{x}_n =$ any multiple of $(1, 1, 1)$.

3. Give an example of a matrix A with the least number of rows and a nonzero vector $\mathbf{b}$ such that the solutions of $A\mathbf{x} = \mathbf{b}$ form a line in $\mathbf{R}^3$, and all the entries of the matrix A are nonzero. Also find all solutions $\mathbf{x}$.

4. Section 3.5, Problem 2

   Find the largest possible number of independent vectors among

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \; \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \; \mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \; \mathbf{v}_4 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \; \mathbf{v}_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \; \mathbf{v}_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}.$$

5. Suppose the rows of an m by n matrix A are linearly independent.

   (a) What is the relation of m and n?

   (b) Is $A\mathbf{x} = \mathbf{b}$ always consistent (solvable) for any $\mathbf{b}$ in $\mathbf{R}^m$?

   (c) If $A\mathbf{x} = \mathbf{b}$ is solvable, is the solution necessarily unique?

   (d) What are the column space of A and the nullspace of A^T?

6. Section 3.5, Problem 24

   U comes from A by subtracting row 1 from row 3:

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ 1 & 3 & 2 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

   Find bases for the two column spaces. Find bases for the two row spaces. Find bases for the two nullspaces.

7. Section 3.6, Problem 4

   Construct a matrix with the required property or explain why this is impossible.

   (a) Column space contains $\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$, row space contains $\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}2\\5\end{bmatrix}$.

   (b) Column space has basis $\begin{bmatrix}1\\1\\3\end{bmatrix}$, nullspace has basis $\begin{bmatrix}3\\1\\1\end{bmatrix}$.

   (c) Dimension of nullspace = 1+ dimension of left nullspace.

   (d) Left nullspace contains $\begin{bmatrix}1\\3\end{bmatrix}$, row space contains $\begin{bmatrix}3\\1\end{bmatrix}$.

   (e) Row space = column space, nullspace $\neq$ left nullspace.

8. Section 3.6, Problem 11

   A is an m by n matrix of rank r. Suppose there are right sides $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ has *no solution*.

   (a) What are all inequalities ($<$ or $\leqq$) that must be true between m, n, and r?

   (b) How do you know that $A^T\mathbf{y} = \mathbf{0}$ has solutions other than $\mathbf{y} = \mathbf{0}$?

9. Suppose A is a 3 by 2 matrix, and B is a 2 by 5 matrix. How many possible dimensions of the nullspace of AB are there?

10. Section 3.6, Problem 23

    Without multiplying matrices, find bases for the row and column spaces of A:

$$A = \begin{bmatrix}1 & 2\\4 & 5\\2 & 7\end{bmatrix}\begin{bmatrix}3 & 0 & 3\\1 & 1 & 2\end{bmatrix}.$$

    How do you know from these shapes that A is not invertible?