Linear Algebra

Problem Set 5 **2009**

Due Wednesday, 22 April 2009 at 12:00 AM in EE207. Be sure to print your name and student ID on your homework.

1. Section 7.1, Problem 6
   Which of these transformations satisfy $T(\mathbf{v}+\mathbf{w}) = T(\mathbf{v})+T(\mathbf{w})$ and which satisfy $T(c\mathbf{v}) = cT(\mathbf{v})$?
   (a) $T(\mathbf{v}) = \mathbf{v}/\|\mathbf{v}\|$
   (b) $T(\mathbf{v}) = v_1+ v_2+v_3$
   (c) $T(\mathbf{v}) = (v_1, 2v_2, 3v_3)$
   (d) $T(\mathbf{v})$ = largest component of $\mathbf{v}$.
2. An *affine transformation* T: $\mathbf{R}^n \rightarrow \mathbf{R}^m$ has the form $T(\mathbf{x}) = A\mathbf{x}+\mathbf{b}$, with A an m by n matrix and $\mathbf{b}$ in $\mathbf{R}^m$. Show that T is not a linear transformation when $\mathbf{b} \neq \mathbf{0}$.
3. Section 7.1, Problem 10
   A linear transformation from **V** to **W** has an *inverse* from **W** to **V** when the range is all of **W** and the kernel contains only $\mathbf{v} = \mathbf{0}$. Why are these transformations not invertible?
   (a) $T(v_1,v_2) = (v_2, v_2)$ $\quad$ $\mathbf{W} = \mathbf{R}^2$
   (b) $T(v_1,v_2) = (v_1, v_2, v_1+v_2)$ $\quad$ $\mathbf{W} = \mathbf{R}^3$
   (c) $T(v_1,v_2) = v_1$ $\quad$ $\mathbf{W} = \mathbf{R}^1$
4. Section 7.1, Problem 17
   The transformation T that transposes every matrix is definitely linear. Which of these extra properties are true?
   (a) T^2 = identity transformation. ($T^2(M)=T(T(M))$)
   (b) The kernel of T is the zero matrix.
   (c) Every matrix is in the range of T.
   (d) $T(M) = -M$ is impossible.
5. Section 7.1, Problem 18
   Suppose $T(M) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} [M] \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$. Find a matrix with $T(M) \neq 0$. Describe all matrices with $T(M) = 0$ (the kernel of T) and all output matrices $T(M)$ (the range of T).
6. A linear transformation T: $\mathbf{R}^2 \rightarrow \mathbf{R}^2$ first reflects points through the x-axis and then reflects points through the y-axis. Show that T can also be described as a linear transformation that rotates points about the origin. What is the angle of that rotation?

7. Suppose T: $\mathbf{R}^n \rightarrow \mathbf{R}^m$ is a linear transformation.
   (a) If T map $\mathbf{R}^n$ onto $\mathbf{R}^m$, what is the relationship between m and n?
   (b) If T is one-to-one, what can you say about m and n?
8. Suppose T: $\mathbf{R}^2 \rightarrow \mathbf{R}^4$, $T(\mathbf{e}_1) = (1, 2, 3, 4)$ and $T(\mathbf{e}_2) = (5, 5, 0, 0)$, where $\mathbf{e}_1=(1, 0)$ and $\mathbf{e}_2=(0,1)$. Find the standard matrix A of T, i.e., $T(\mathbf{x})=A\mathbf{x}$. Also, find $T(\mathbf{x})$, where $\mathbf{x}=(2, -3)$.
9. Let $\mathbf{w}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $\mathbf{w}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$, and $B = \{\mathbf{w}_1, \mathbf{w}_2\}$. Find the coordinate vector $[\mathbf{x}]_B$ of $\mathbf{x}$ with respect to basis B. Find $\mathbf{y}$ such that $[\mathbf{y}]_B=(2,4)$. What is the change-of-coordinate matrix from B to the standard basis in $\mathbf{R}^2$?
10. Use coordinate vectors to verify that the polynomials $1+2t^2$, $4+t+5t^2$, and $3+2t$ are *linearly dependent* in $\mathbf{P}^2$. (Identify the coordinate vectors of these polynomials relative to the standard basis, and then show that these vectors are linear dependent.)