Linear Algebra
Problem Set 6 **2009**

Due Wednesday, 29 April 2009 at 10:00 AM in EE207. Be sure to print your name and student ID on your homework.

1. Suppose T is a linear transformation from $\mathbf{R}^3$ to $\mathbf{R}^3$ and

$$T\left(\begin{bmatrix}1\\0\\1\end{bmatrix}\right)=\begin{bmatrix}-1\\0\\1\end{bmatrix},\ T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right)=\begin{bmatrix}1\\0\\-1\end{bmatrix},\ T\left(\begin{bmatrix}1\\0\\-1\end{bmatrix}\right)=\begin{bmatrix}1\\-2\\1\end{bmatrix}.$$

   (a) Find the matrix of the linear transformation with respect to the standard basis.
   (b) Find the matrix of the linear transformation with respect to the following basis:

$$B=\left\{\begin{bmatrix}1\\0\\1\end{bmatrix},\ \begin{bmatrix}0\\1\\0\end{bmatrix},\ \begin{bmatrix}1\\0\\-1\end{bmatrix}\right\}.$$

2. Suppose L is a linear transformation from $\mathbf{P}^2$ to $\mathbf{P}^3$, described by

$$L(f(x)) = xf(x) + f'(x).$$

   Find a basis for the range of L. (The basis you choose can be represented as a set of coordinate vectors with respect to the standard basis $\{1, x, x^2, x^3\}$.)
3. An affine transformation T: $\mathbf{R}^2\rightarrow\mathbf{R}^2$ has the form $T(\mathbf{x}) = A\mathbf{x}+\mathbf{b}$, with A an 2 by 2 matrix and $\mathbf{b}$ in $\mathbf{R}^2$. If T is invertible, show the following statements:
   (a) $T^{-1}(\mathbf{x}) = A^{-1}\mathbf{x} - A^{-1}\mathbf{b}$.
   (b) Affine transformations map parallel straight lines to parallel straight lines.
4. Let L be the linear transformation that rotates vectors in $\mathbf{R}^2$ by 45° in the counterclockwise direction. Will it be possible to find a basis B so that the matrix of L with respect to B is the identity matrix, i.e., $[L]_B = I$?
5. Let $\mathbf{b}_1=\begin{bmatrix}1\\-3\end{bmatrix}$, $\mathbf{b}_2=\begin{bmatrix}-2\\4\end{bmatrix}$, $\mathbf{c}_1=\begin{bmatrix}-7\\9\end{bmatrix}$, $\mathbf{c}_2=\begin{bmatrix}-5\\7\end{bmatrix}$, and consider the bases for $\mathbf{R}^2$ given by $B = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $C = \{\mathbf{c}_1, \mathbf{c}_2\}$.
   (a) Find the change-of-coordinates matrix M from B to C. Note that $[\mathbf{v}]_C = M[\mathbf{v}]_B$, where $[\mathbf{v}]_C$ denotes coordinate vector of $\mathbf{v}$ with respect to basis C.
   (b) Suppose the matrix of the linear transformation T with respect to basis B is $[T]_B=\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$. What is the matrix of T with respect to basis C?
6. Section 4.1, Problem 17
   If S is the subspace of $\mathbf{R}^3$ containing only the zero vector, what is $S^{\perp}$? If S is

spanned by (1, 1, 1), what is $S^{\perp}$? If S is spanned by (2, 0, 0) and (0, 0, 3), what is $S^{\perp}$?

7. Section 4.1, Problem 28

   Why is each of these statements false?

   (a) (1, 1, 1) is perpendicular to (1, 1, -2) so the planes $x+y+z=0$ and $x+y-2z=0$ are orthogonal subspaces.

   (b) The subspace spanned by (1, 1, 0, 0, 0) and (0, 0, 0, 1, 1) is the orthogonal complement of the subspaces spanned by (1, -1, 0, 0, 0) and (2, -2, 3, 4, -4).

   (c) If two subspaces meet only in the zero vector, the subspaces are orthogonal.

8. Section 4.1, Problem 30

   Suppose A is 3 by 4 and B is 4 by 5 and AB=0. Prove that rank(A)+rank(B) $\leqq$ 4.

9. Section 4.2, Problem 13

   Suppose A is the 4 by 4 identity matrix with its last column removed. A is 4 by 3. Project **b** = (1, 2, 3, 4) onto the column space of A. What shape is the projection matrix P and what is P?

10. Section 4.2, Problem 19

    To find the projection matrix onto the plane $x-y-2z = 0$, choose two vectors in that plane and make them the columns of A. The plane should be the column space. Then compute $P = A(A^T A)^{-1} A^T$.