**Linear Algebra**

**Problem Set 7** **2009**

Due Wednesday, 6 May 2009 at 10:00 AM in EE207. Be sure to print your name and student ID on your homework.

1. Suppose $L$ is the line $ax+by=0$, and $P=(x, y)$ is a point on the $xy$-plane. If $Q=(x_1, y_1)$ is the foot of the perpendicular from $P$ to $L$, find a 2 by 2 matrix $A$ such that

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = A \begin{bmatrix} x \\ y \end{bmatrix}.$$

2. Suppose W is the subspace spanned by $\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}$. Find the point in W that is closest to $\begin{bmatrix} 4 \\ -2 \\ -3 \end{bmatrix}$.

3. Section 4.3, Problem 1
   With $b = 0, 8, 8, 20$ at $t = 0, 1, 3, 4$, set up and solve the normal equations $A^T A\hat{\mathbf{x}} = A^T\mathbf{b}$. For the best straight line in figure 4.9a, find its four heights $p_i$ and four errors $e_i$. What is the minimum value $E = e_1^2 + e_2^2 + e_3^2 + e_4^2$ ?

4. Section 4.3, Problem 17
   Write down three equations for the line $b = C + Dt$ to go through $b = 7$ at $t = -1$, $b = 7$ at $t = 1$, and $b = 21$ at $t = 2$. Find the least squares solution $\hat{\mathbf{x}} = (C, D)$ and draw the closest line.

5. Section 4.4, Problem 4
   Give an example of each of the following:
   (a) A matrix $Q$ that has orthonormal columns but $QQ^T \neq I$.
   (b) Two orthogonal vectors that are not linearly independent.
   (c) An orthonormal basis for $\mathbf{R}^4$, where every component is $\frac{1}{2}$ or $-\frac{1}{2}$.

6. Suppose $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$. Find the solution of $A\mathbf{x} = \mathbf{b}$ and $\mathbf{x}$ has the minimum length $\|\mathbf{x}\|$. Is such an $\mathbf{x}$ always in the row space of $A$?

7. Section 4.4, Problem 15
   (a) Find orthonormal vectors $\mathbf{q}_1$, $\mathbf{q}_2$, $\mathbf{q}_3$ such that $\mathbf{q}_1$, $\mathbf{q}_2$ span the column space of

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 4 \end{bmatrix}.$$

   (b) Which of the four fundamental subspaces contains $\mathbf{q}_3$?
   (c) Solve $A\mathbf{x} = (1, 2, 7)$ by least squares.
8. Section 4.4, Problem 23
   Find $\mathbf{q}_1$, $\mathbf{q}_2$, $\mathbf{q}_3$ (orthonormal) as combinations of **a**, **b**, **c** (independent columns of $A$). Then write $A$ as $QR$:

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 5 \\ 0 & 3 & 6 \end{bmatrix}.$$

9. Section 4.4, Problem 24
   (a) Find a basis for the subspace $S$ in $\mathbf{R}^4$ spanned by all solutions of
$$x_1 + x_2 + x_3 - x_4 = 0.$$
   (b) Find a basis for the orthogonal complement $S^\perp$.
   (c) Find $\mathbf{b}_1$ in $S$ and $\mathbf{b}_2$ in $S^\perp$ so that $\mathbf{b}_1 + \mathbf{b}_2 = \mathbf{b} = (1, 1, 1, 1)$.
10. Section 4.4, Problem 34
   $Q = I - 2\mathbf{u}\mathbf{u}^T$ is a reflection matrix when $\mathbf{u}^T\mathbf{u} = 1$.
   (a) Show that $Q\mathbf{u} = -\mathbf{u}$. The mirror is perpendicular to **u**.
   (b) Find $Q\mathbf{v}$ when $\mathbf{u}^T\mathbf{v} = 0$. The mirror contains **v**. It reflects to itself.