Linear Algebra **Spring 2009**

Solutions to Problem Set 1

1. 2.1 (18)

$P=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$ produces (y,z,x) and Q= $P^T=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$ recovers (x,y,z).

2. 2.1 (20)

$E=\begin{bmatrix}1&0&0\\0&1&0\\1&0&1\end{bmatrix}$, $E^{-1}=\begin{bmatrix}1&0&0\\0&1&0\\-1&0&1\end{bmatrix}$, Ev=(3,4,8) , $E^{-1}Ev$=(3,4,5).

3. 2.3 (3)

$E_{21}=\begin{bmatrix}1&0&0\\-4&1&0\\0&0&1\end{bmatrix}$,$E_{31}=\begin{bmatrix}1&0&0\\0&1&0\\2&0&1\end{bmatrix}$,$E_{32}=\begin{bmatrix}1&0&0\\0&1&0\\0&-2&1\end{bmatrix}$

$M=E_{32}E_{31}E_{21}=\begin{bmatrix}1&0&0\\-4&1&0\\10&-2&1\end{bmatrix}$.

4.

$$\begin{bmatrix}a&0&b&2\\a&a&4&4\\0&a&2&b\end{bmatrix}\xrightarrow{E_{21}}\begin{bmatrix}a&0&b&2\\0&a&4-b&2\\0&a&2&b\end{bmatrix}\xrightarrow{E_{32}}\begin{bmatrix}a&0&b&2\\0&a&4-b&2\\0&0&b-2&b-2\end{bmatrix}$$

(a) if a ≠ 0 and b ≠ 2, the linear system has a unique solution.

(b) if a = 0 and b ≠ 2, system has no solution.

(c) if b = 2, system has infinite many solutions.

5. 2.3 (30)

$E=\begin{bmatrix}1&0&0&0\\-1&1&0&0\\0&-1&1&0\\0&0&-1&1\end{bmatrix}$, what we do is "$row2-row1;\ row3-row2;\ row4-row3$".

6. 2.4 (7)

(a). True

(b). False, if $A=\begin{bmatrix}1&0&1\\1&1&0\\1&1&1\end{bmatrix}$, $B=\begin{bmatrix}1\\1\\1\end{bmatrix}$, $AB=\begin{bmatrix}2\\2\\3\end{bmatrix}$ (row 1 and 3 are different).

(c). True

(d). False, if $A=\begin{bmatrix}1&2\\0&0\end{bmatrix}$, $B=\begin{bmatrix}1&0\\3&0\end{bmatrix}$, $(AB)^2=\begin{bmatrix}49&0\\0&0\end{bmatrix}$, but $A^2B^2=\begin{bmatrix}7&0\\0&0\end{bmatrix}$.

7. 2.4 (11)

(a). BA=4A=4IA $\therefore$ B=4I

(b). BA=4B for any A $\therefore$ B=0

(c). $B=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$

(d). $B=\begin{bmatrix}1&0&0\\1&0&0\\1&0&0\end{bmatrix}$

8. 2.4 (16)

By linearity (AB)c agrees with A(Bc). Also for all other columns of C.

9.

(a). $X=\begin{bmatrix}1&1&1\\1&1&0\\1&0&0\end{bmatrix}$ (用行向量的線性組合思考)

(b). $X=\begin{bmatrix}0&0&0&3\\0&1&1&0\\0&2&0&0\\1&0&0&0\end{bmatrix}$ (用列向量的線性組合思考)

10. 2.4 (34)

Let $x_1=\begin{bmatrix}1\\1\\1\end{bmatrix}, x_2=\begin{bmatrix}0\\1\\1\end{bmatrix},\ x_3=\begin{bmatrix}0\\0\\1\end{bmatrix}$

b= $\begin{bmatrix}3\\5\\8\end{bmatrix}=3\begin{bmatrix}1\\0\\0\end{bmatrix}+5\begin{bmatrix}0\\1\\0\end{bmatrix}+8\begin{bmatrix}0\\0\\1\end{bmatrix}=3\text{A}x_1+5\text{A}x_2+8\text{A}x_3=\text{A}(3x_1+5x_2+8x_3)$

$\therefore x=3x_1+5x_2+8x_3=\begin{bmatrix}3\\8\\16\end{bmatrix}$

AX=A $\begin{bmatrix}1&0&0\\1&1&0\\1&1&1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

利用觀察列向量的線性組合 可求得A= $\begin{bmatrix}1&0&0\\-1&1&0\\0&-1&1\end{bmatrix}$.