Linear Algebra Spring2009

Solution to Problem Set 2

1. 2.5(7)

   (a) In Ax=$(1,0,0)$,equation 1+equation 2-equation 3 is 0≠1

   (b) The right sides must satisfy $b_1 + b_2 = b_3$

   (c) Row 3 becomes a row of zeros ,no third pivot.

2. 2.5(10)

   (a) $A^{-1} = \begin{bmatrix} 0 & 0 & 0 & 1/5 \\ 0 & 0 & 1/4 & 0 \\ 0 & 1/3 & 0 & 0 \\ 1/2 & 0 & 0 & 0 \end{bmatrix}$

   (b) $B^{-1} = \begin{bmatrix} 3 & -2 & 0 & 0 \\ -4 & 3 & 0 & 0 \\ 0 & 0 & 6 & -5 \\ 0 & 0 & -7 & 6 \end{bmatrix}$ (invert each block)

3. 2.5(32)

   $A^{-1} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ The 5 by 5 A^{-1}also has 1's on the diagonal and super-diagonal.

4.

$$
\begin{aligned}
A \cdot A^{-1} &= (I_n - UV)\left(I_n + U(I_m - VU)^{-1}V\right) \\
&= I_n + U(I_m - VU)^{-1}V - UV - UVU(I_m - VU)^{-1}V \\
&= I_n + U\left[(I_m - VU)^{-1} - I_m - VU(I_m - VU)^{-1}\right]V \\
&= I_n + U\left[(I_m - VU)(I_m - VU)^{-1} - I_m\right]V \\
&= I_n + U\left[I_m - I_m\right]V \\
&= I_n
\end{aligned}
$$

5.

方法(一):用高斯消去法比對細數即可

方法(二):

$$令A=\begin{bmatrix}1&2&3\\2&5&3\\1&0&8\end{bmatrix},v_1=\begin{bmatrix}1\\a\\6\end{bmatrix},v_2=\begin{bmatrix}b\\0\\c\end{bmatrix}$$

因此原矩陣可表示為 $\begin{bmatrix}|&|&|\\A&v_1&v_2\\|&|&|\end{bmatrix}\xrightarrow{A^{-1}}\begin{bmatrix}|&-2&0\\I&d&-1\\|&1&e\end{bmatrix}$

這代表說 $A^{-1}\cdot v_1=\begin{bmatrix}-2\\d\\1\end{bmatrix}$, $A^{-1}\cdot v_2=\begin{bmatrix}0\\-1\\e\end{bmatrix}$

換個表示法 $A\cdot\begin{bmatrix}-2\\d\\1\end{bmatrix}=v_1$, $A\cdot\begin{bmatrix}0\\-1\\e\end{bmatrix}=v_2$

$\therefore\begin{bmatrix}1&2&3\\2&5&3\\1&0&8\end{bmatrix}\begin{bmatrix}-2\\d\\1\end{bmatrix}=\begin{bmatrix}1\\a\\6\end{bmatrix},\begin{bmatrix}1&2&3\\2&5&3\\1&0&8\end{bmatrix}\begin{bmatrix}0\\-1\\e\end{bmatrix}=\begin{bmatrix}b\\0\\c\end{bmatrix}$

可解出$a=-1,b=3,c={}^{40}/_{3},d=0,e={}^{5}/_{3}$

6.

$$B=(I+A)^{-1}(I-A)$$

$$(I+A)B=(I-A)$$

$$B+AB+A=I$$

$$(I+B)+A(I+B)=2I$$

$$(I+B)(I+A)=2I$$

$$\therefore(I+B)^{-1}=\tfrac{1}{2}(I+A)=\begin{bmatrix}1&0&0&0\\-1&2&0&0\\0&-2&3&0\\0&0&-3&4\end{bmatrix}$$

7. 2.6(13)

$$\begin{bmatrix}a&a&a&a\\a&b&b&b\\a&b&c&c\\a&b&c&d\end{bmatrix}=\begin{bmatrix}1&&&\\1&1&&\\1&1&1&\\1&1&1&1\end{bmatrix}\begin{bmatrix}a&a&a&a\\&b-a&b-a&b-a\\&&c-b&c-b\\&&&d-c\end{bmatrix}$$

Need $a\neq0,\ b\neq a,\ c\neq b,\ d\neq c$

8. 2.6(18)

   (a) Multipy $LDU = L_1D_1U_1$by inverses to get $L_1^{-1}LD = D_1U_1U^{-1}$.The left side is lower triangular,the right side is upper triangular⇒both sides are diagonal.

   (b) Since L, U, L_1, U_1have diagonals of 1's we get $D = D_1$.Then $L_1^{-1}L$ is I and U_1U^{-1} is I.

9. 2.7(18)

   (a) $\because A$為對稱
   $\therefore$ 只有一半的$entries$加上對角軸可自由選擇
   $\Rightarrow 5+4+3+2+1 = 15$ independent entries if $A = A^T$.

   (b) L has 10 and D has 5; total 15 in LDL^T.

   (c) Zero diagonal if $A^T = -A$,leaving 4+3+2+1=10 choise.

10.
首先,等號兩邊都取轉置

$$\begin{bmatrix} -1 & 1 & 3 \\ 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} X^T = \begin{bmatrix} 1 & -3 \\ 2 & 1 \\ 0 & 5 \end{bmatrix}$$

接下來寫成增廣矩陣的模式,開始用消去法

$$\begin{bmatrix} -1 & 1 & 3 & 1 & -3 \\ 0 & 1 & 1 & 2 & 1 \\ 1 & 0 & -1 & 0 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 1 & 3 & 1 & -3 \\ 0 & 1 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & -3 & -1 & 3 \\ 0 & 1 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1 & 2 \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} 1 & -1 & -3 & -1 & 3 \\ 0 & 1 & 1 & 2 & 1 \\ 0 & 0 & 1 & -1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 0 & -4 & 6 \\ 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 1 & -1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -1 & 6 \\ 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 1 & -1 & 1 \end{bmatrix}$$

$$\therefore X^T = \begin{bmatrix} -1 & 6 \\ 3 & 0 \\ -1 & 1 \end{bmatrix} \rightarrow X = \begin{bmatrix} -1 & 3 & -1 \\ 6 & 0 & 1 \end{bmatrix}$$