Linear Algebra Spring2009

Solution to Problem Set 3

1.

(a) $\because AA = A$ 兩邊同乘A^{-1}

$\Rightarrow A^{-1}AA = A^{-1}A$

$\Rightarrow A = I$

(b)

guess $B = (I - cA)^{-1} = I + kA$

$B(I - cA) = (I + kA)(I - cA)$

$= I + kA - cA - ckAA$

$= I + (k - c - ck)A$

if $k - c - ck = 0$, then $B(I - cA) = I$ 即$B = (I - cA)^{-1}$

$\Rightarrow k = \frac{c}{1-c} \quad , c \neq 1$

$\therefore B = I + \frac{c}{1-c}A \ , c \neq 1$

2.

$\because E^2 = nE$

we guess $(I - E)^{-1} = I + kE$

$\therefore (I - E)(I + kE) = I - E + kE - kEE = I - E + kE - nkE = I$

$\therefore k - 1 - nk = 0 \Rightarrow k = \frac{1}{1-n}$

$$(I - E)^{-1} = \frac{1}{1-n}\begin{bmatrix} 2-n & 1 & \cdots & 1 \\ 1 & \ddots & & \vdots \\ \vdots & & \ddots & 1 \\ 1 & \cdots & 1 & 2-n \end{bmatrix}$$

3. 2.4(35)

(a) The (2,2) block $S = D - CA^{-1}B$ is the Schur complement

(b) $X = \begin{bmatrix} I & A^{-1}B \\ 0 & I \end{bmatrix}$

4.

(a) $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} I & 0 \\ CA^{-1} & I \end{bmatrix} \begin{bmatrix} A & 0 \\ 0 & D - CA^{-1}B \end{bmatrix} \begin{bmatrix} I & A^{-1}B \\ 0 & I \end{bmatrix}$

(b) $\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} I & -A^{-1}B \\ 0 & I \end{bmatrix} \begin{bmatrix} A^{-1} & 0 \\ 0 & (D - CA^{-1}B)^{-1} \end{bmatrix} \begin{bmatrix} I & 0 \\ -CA^{-1} & I \end{bmatrix}$

5. 3.1(5)

   (a) Yes

   (b) No, 沒通過原點

   (c) No, ex:$(1,1,0) \in V, (0,0,1) \in V$ $but (1,1,0) + (0,0,1) = (1,1,1) \notin V$

   (d) Yes

   (e) Yes

   (f) No, ex:$(1,2,3) \in V$ but $(-1,-2,-3) \notin V$

6. 3.1(4)

   The zero vector in M is $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$; $\frac{1}{2}A = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$ and $-A = \begin{bmatrix} -2 & 2 \\ -2 & 2 \end{bmatrix}$.

   The smallest subspace containing A consists of all matrices cA.

7. 3.1(18)

   (a) True

   (b) True

   (c) False,because not include $\mathbf{0}$

8. $b2 = b3$

9. The extra column b enlarges the column space unless b is already in the column space of A:

   $\begin{bmatrix} A & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $(larger\ column\ space)$ $(no\ solution\ to\ Ax = b)$ $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ $(b\ already\ in\ column\ space)$ $(Ax = b\ has\ a\ solution)$

10. 3.1(27)

    (a) False ,because not include $\mathbf{0}$.

    (b) True

    (c) True

    (d) False ,ex:$A = I$, $A - I = 0$ not equals the column space of A.