Linear Algebra Spring2009

Solution to Problem Set 4

1. 3.4(1)

$$\begin{bmatrix} 2 & 4 & 6 & 4 & b_1 \\ 2 & 5 & 7 & 6 & b_2 \\ 2 & 3 & 5 & 2 & b_3 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 4 & 6 & 4 & b_1 \\ 0 & 1 & 1 & 2 & b_2 - b_1 \\ 0 & -1 & -1 & -2 & b_3 - b_1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 4 & 6 & 4 & b_1 \\ 0 & 1 & 1 & 2 & b_2 - b_1 \\ 0 & 0 & 0 & 0 & b_3 + b_2 - 2b_1 \end{bmatrix}$$

$Ax = b$ has a solution when $b_3 + b_2 - 2b_1 = 0$;the column space contains all combinations of $(2, 2, 2)$and$(4, 5, 3)$which is the plane $b_3 + b_2 - 2b_1 = 0$;the nullspace contains all combinations of $s_1 = (-1, -1, 1, 0)$and $s_2 = (2, -2, 0, 1)$;

$x_{complete} = x_p + c_1 s_1 + c_2 s_2$;

$\begin{bmatrix} R & d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & -2 & 4 \\ 0 & 1 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ gives the particular solution $x_p = (4, -1, 0, 0)$.

2. 3.4(10)

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} x = \begin{bmatrix} 2 \\ 4 \end{bmatrix}.$$

3.

$x_{complete}$ form a line in $\mathrm{R}^3 \Rightarrow x_{complete} = x_p + c_1 s_1$

$\Rightarrow dimN(A) = 1$

$\Rightarrow rank(A) = 2$

$\because$ 題目說要最少的row $\Rightarrow$ 取row為2 & entries of A 不可以為0

Let $Ax = b \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$x_{complete} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

4. 3.5(2)

v_1, v_2, v_3 are independent. All six vectors are on the plane $(1, 1, 1, 1) \cdot v = 0$

$\therefore dim(v) = 3$

$\therefore$so no four of these six vectors can be independent.

5.

(a) $m \leq n$

(b) Yes,$\because$ $C(A) \in R^m$& $dimC(A) = m$ $\therefore$for any $b \in R^m$ must in $C(A)$.

(c) No, $\because$dim N(A)$\neq 0$

(d) $C(A) = R^m$; $N(A^T) = 0$

6. 3.5(24)

   Columns 1 and 2 are bases for the (different) column spaces;

   rows 1 and 2 are bases for the (equal) row spaces;

   (1,-1,1) is a basis for the (equal) nullspaces.

7. 3.6(4)

   (a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$

   (b) Impossible: $r + (n - r)$ must be 3

   (c) $\begin{bmatrix} 1 & 1 \end{bmatrix}$

   (d) $\begin{bmatrix} -9 & -3 \\ 3 & 1 \end{bmatrix}$

   (e) Impossible: Row space=column space requires m=n.Then $m - r = n - r$.

8. 3.6(11)

   (a) No solution means that $r < m$.Always $r \leq n$.Can't compare m and n.

   (b) If $m - r > 0$,the left nullspace contains a nonzero vector.

9.

   $\because C(AB) \in C(A)$

   $\Rightarrow$r=rank AB$\leq$rank A$\leq$2

   $\Rightarrow$dim N(AB)=5 $- r\geq$ 3

   $\therefore$dim N(AB)=3 or 4 or 5

10. 3.6(23)

    Row space basis $(3, 0, 3), (1, 1, 2)$;column space basis $(1, 4, 2), (2, 5, 7)$;rank is only 2.