Linear Algebra Spring2009

Solution to Problem Set 5

1. 7.1(6)

   (b) and (c) are linear (d) satisfies T(cv)=cT(v)

2.

   if $b \neq 0$

   $T(v+w) = A(v+w) + b = Av + Aw + b \neq (Av+b) + (Aw+b) = T(v) + T(w)$

   $T(cv) = cAv + b \neq cT(v)$

   $\therefore T$ is not a linear transformation.

3. 7.1(10)

   (a) $T(1,0) = 0$

   (b) $(0,0,1)$ is not in the range.

   (c) $T(0,1) = 0$

4. 7.1(17)

   (a) True

   (b) True

   (c) True

   (d) False

5. 7.1(18)

   Let $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

   $T(M) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$

   $\therefore b = 0, M = \begin{bmatrix} a & 0 \\ c & d \end{bmatrix}$ in the kernel.$T(M) = 0$

   $b \neq 0, T(M) = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$

   the range of T is $\begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$ and 0.

6.

T can reflects points through the x-axis and y-axis

$$T(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} -x \\ -y \end{bmatrix} \Rightarrow T = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

we know rotate transformation $T = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$\therefore \theta = 180°$

7.

(a) $n \geq m$

(b) $m \geq n$

8.

$$A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 2 & 5 \\ 3 & 0 \\ 4 & 0 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 1 & 5 \\ 2 & 5 \\ 3 & 0 \\ 4 & 0 \end{bmatrix}$$

$$T(\begin{bmatrix} 2 \\ -3 \end{bmatrix}) = A \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 2 & 5 \\ 3 & 0 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} - 3 \begin{bmatrix} 5 \\ 5 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -13 \\ -11 \\ 6 \\ 8 \end{bmatrix}$$

9.

(a)

$$\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} [x]_B = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$$

$$[x]_B = inv(\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}) \cdot \begin{bmatrix} 5 \\ -4 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} [y]_B = \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

(b)

standard basis in $R^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} [x]_B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} [x]$$

$$\Rightarrow [x] = \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} [x]_B$$

$\Rightarrow$change-of-coordinate matrix is $\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$.

10. Let standard basis of $P^2$is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\left[1+2t^2\right]_s = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$\left[4+t+5t^2\right]_s = \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}$$

$$[3+2t]_s = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$$

Let $A = \begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 2 \\ 2 & 5 & 0 \end{bmatrix} \Rightarrow if$ rank(A)$\neq$3 ,they are linear dependent.

$$\begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 2 \\ 2 & 5 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 2 \\ 0 & -3 & -6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore rank(A) = 2 \Rightarrow$they are linearly dependent.