Linear Algebra Spring2009

Solution to Problem Set 6

1.

(a)

$$A\begin{bmatrix}1&0&1\\0&1&0\\1&0&-1\end{bmatrix}=\begin{bmatrix}-1&1&1\\0&0&-2\\1&-1&1\end{bmatrix}\Rightarrow A=\begin{bmatrix}0&1&-1\\-1&0&1\\1&-1&0\end{bmatrix}$$

(b)

Let $B=\begin{bmatrix}1&0&1\\0&1&0\\1&0&-1\end{bmatrix}$

$A^{'}=B^{-1}AB=\begin{bmatrix}0&0&1\\0&0&-2\\-1&1&0\end{bmatrix}$

2.

$1\overset{L}{\rightarrow}x$; $x\overset{L}{\rightarrow}1+x^2$; $x^2\overset{L}{\rightarrow}2x+x^3$

$$\therefore A\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}0&1&0\\1&0&2\\0&1&0\\0&0&1\end{bmatrix}\Rightarrow A=\begin{bmatrix}0&1&0\\1&0&2\\0&1&0\\0&0&1\end{bmatrix}$$

$\therefore$the basis for the range of L is $\begin{bmatrix}0\\1\\0\\0\end{bmatrix},\begin{bmatrix}1\\0\\1\\0\end{bmatrix},\begin{bmatrix}0\\2\\0\\1\end{bmatrix}$

3.

(a)we need to show that $T^{-1}(T(x))=x$& $T(T^{-1}(x))=x$

$T^{-1}(T(x))=A^{-1}(Ax+b)-A^{-1}b=x+A^{-1}b-A^{-1}b=x$

$T(T^{-1}(x))=A(A^{-1}x-A^{-1}b)+b=x-b+b=x$

(b)

Let L_1 : 法向量m_1 L_2 : 法向量m_2

$T(m_1)=Am_1+b$ $T(m_2)=Am_2+b$

if $L_1//L_2\Rightarrow m_1=m_2\Rightarrow T(m_1)=T(m_2)$

轉換後的法向量仍然相同$\Rightarrow L_1^{'}//L_2^{'}$

4.

No,it is not possible to find the basis B.

$\because L = B\,[L]_B\,B^{-1}$

if $[L]_B = I$,then $L = BB^{-1} = I$

but L is rotate matrix(45 degrees).

5. (a)

$$\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix} [v]_B = \begin{bmatrix} -7 & -5 \\ 9 & 7 \end{bmatrix} [v]_C$$

$$\Rightarrow [v]_C = \begin{bmatrix} 2 & -1.5 \\ -3 & 2.5 \end{bmatrix} [v]_B$$

$$\Rightarrow M = \begin{bmatrix} 2 & -1.5 \\ -3 & 2.5 \end{bmatrix}$$

(b)

$$[T]_C = M[T]_B M^{-1} = \begin{bmatrix} 13 & 8 \\ -18 & -11 \end{bmatrix}$$

6. 4.1(17)

If S is the subspace of R^3 containing only the zero vector,then $S^{\perp}$ is R^3 .

If S is spanned by $(1,1,1)$,then $S^{\perp}$is spanned by $(1,-1,0)$ and $(1,0,-1)$,which is $null$space of S.

If S is spanned by $(2,0,0)$and $(0,0,3)$,then $S^{\perp}$is spanned by $(0,1,0)$,which is $null$space of S.

7. 4.1(28)

   (a) $(1,-1,0)$is in both planes.Normal vectors are perpendicular,but planes still intersect!

   (b) Need *three* orthogonal vectors to span the whole orthogonal complement.

   (c) Lines can meet without being orthogonal.

8. 4.1(30)

When $AB = 0$,the column space of B is contained in the nullspace of A.Therefore the dimension of $C(B) \leq dimension$of $N(A)$.This means $rank(B) \leq 4 - rank(A)$.

9. 4.2(13)

$$p = (1,2,3,0).\ P = square\ matrix = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

10. 4.2(19)

For any choice,say $(1,1,0)$ and$(2,0,1)$,the matrix P is $\begin{bmatrix} 5/6 & 1/6 & 1/3 \\ 1/6 & 5/6 & -1/3 \\ 1/3 & -1/3 & 1/3 \end{bmatrix}$.