Linear Algebra Spring2009

Solution to Problem Set 8

1. 5.1(16)

   A singular rank one matrix has det=0 ;

   Also det K=0. $\because det(K) = det(K^T)$, but $K^T = -K$ $\therefore det(K) = -det(K) = 0$.

2. 5.1(18)

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} = (b-a)(c-a)\begin{vmatrix} 1 & b+a \\ 1 & c+a \end{vmatrix} = (b-a)(c-a)(c-b).$$

3. 5.1(28)

   (a) True: $det(AB) = det(A)det(B) = 0$.

   (b) False: may exchange rows.

   (c) False: $A = 2I$ and $B = I$.

   (d) True: $det(AB) = det(A)det(B) = det(BA)$.

4.

   $\because P^2 = P \Rightarrow det(P)det(P) = det(P)$

   $\therefore det(P) = 0$ or 1.

   $\because QQ^T = I \Rightarrow det(Q)det(Q^T) = det^2(Q) = 1$

   $\therefore det(Q) = 1$ or -1.

5. 5.1(34)

   Reduce B to $[row3\,;row2\,;row1]$.Then $det(B) = -6$.

6. 5.2(15)

   (a) $C_1 = 0$, $C_2 = -1$, $C_3 = 0$, $C_4 = 1$

   (b) $C_n = -C_{n-2}$ by cofactors of row 1 then cofactors of column 1.

   Therefore $C_{10} = -C_8 = C_6 = -C_4 = -1$.

7. 5.2(27)

   Problem 25 gives det $\begin{bmatrix} I & 0 \\ -CA^{-1} & I \end{bmatrix} = 1$,so det $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = |A|\left|D - CA^{-1}B\right|$which is $\left|AD - ACA^{-1}B\right|$.

   If $AC = CA$,this is $\left|AD - CAA^{-1}B\right| = |AD - CB|$.

8.

$\because rank(Q) = rank(Q^TQ)$ and $rank(Q^T) = rank(QQ^T)$

$\therefore rank(Q^TQ) = rank(QQ^T) = 3$

$\therefore det(Q^TQ) = I_3 = 1$ and $det(QQ^T) = 0$ (because it is 5×5 matrix,but rank only 3).

9.

we know that $A^{-1} = \frac{C^T}{det(A)}$ if $det(A) \neq 0$.

multiply A on both side,we get $det(A)I = AC^T$.This is correct for every A.

$\because AC^T = det(A) \cdot I_N = 0(\because A$ is singular)

$\therefore$row of marix C is in null space of A.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}, C^T = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow N(A) = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}.$$

10. $AC^T = (detA)I_N$

$\Rightarrow (detA)(detC^T) = (detA)^n$

$\Rightarrow detC = detC^T = (detA)^{n-1}$