Linear Algebra
Problem Set 1 **2010**

Due Wednesday, 3 March 2010 at 10:00 AM in EE105. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (20pts) A cubic polynomial $f(t) = a_3t^3 + a_2t^2 + a_1t + a_0$ in t with real coefficients has the value 9 at 2, the value 32 at 3 and the derivative $f'(t)$ has the value 0 at 1 and the value 12 at 2. Find $f(t)$. (Hint: There are two obvious orderings of the variables, one of which gives much easier equation to solve than the others.)

2. (15pts) Let $A = \begin{bmatrix} 2 & -2 & 1 \\ 4 & -3 & -1 \\ -6 & 7 & -6 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ -2 \\ 2t-1 \end{bmatrix}$, where t is a real number. For which values of t does the system of linear equations $A\mathbf{x} = \mathbf{b}$ have a solution?

3. (20pts) The three types of elementary row operations are not independent: the row exchange operation can be accomplished by a sequence of the other two types of row operations, namely, replacing (i.e., subtracting $k \times$ row$i$ from row$j$) and scaling (i.e., multiplying row$i$ by k, $k \neq 0$). Apply a sequence of replacing and scaling operations to the identify matrix I to obtain the permutation matrix

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

4. (15pts) Section 2.3, Problem 31
   Find elimination matrices E_{21} then E_{32} then E_{43} to change K into U:
$$E_{43}E_{32}E_{21}\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & -1 \\ 0 & 0 & 0 & 5/4 \end{bmatrix}.$$
   Apply those three steps to the identity matrix I, to multiply $E_{43}E_{32}E_{21}$.

5. (20pts) Consider the following three systems where the coefficients are the same for each system, but the right-hand sides are different:

$$\begin{array}{l|l|l} 4x - 8y + 5z = 1 & 0 & 0 \\ 4x - 7y + 4z = 0 & 1 & 0 \\ 3x - 4y + 2z = 0 & 0 & 1 \end{array}$$

Solve all three systems at one time by performing Gaussian elimination on an augmented matrix of the form $\left[A\middle|\mathbf{b}_1\middle|\mathbf{b}_2\middle|\mathbf{b}_3\right]$. What is the 3 by 3 matrix X such that $AX=I$?

6. (10pts) Without using Gaussian elimination, solve X by inspection. Identify the size of X first.

(a) $$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 3 & 1 \\ 0 & 4 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & 5 & 3 \\ 2 & 5 & 3 \\ 3 & 5 & 4 \\ 4 & 5 & 5 \end{bmatrix}$$

(b) $$X \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$