Linear Algebra

Problem Set 12 **2010**

Due Wednesday, 9 June 2010 at 10:00 AM in EE102. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (20pts) Section 6.4, Problem 21

   True (with reason) or false (with example). "Orthonormal" is not assumed.

   (a) A matrix with real eigenvalues and eigenvectors is symmetric.

   (b) A matrix with real eigenvalues and orthogonal eigenvectors is symmetric.

   (c) The inverse of a symmetric matrix is symmetric.

   (d) The eigenvector matrix S of a symmetric matrix is symmetric.

2. (20pts) Section 6.4, Problem 23

   Which of these classes of matrices A and B belong to: Invertible, orthogonal, projection, permutation, diagonalizable?

   $$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad B = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

   Which of these factorizations are possible for A and B: LU, QR, $S\Lambda S^{-1}$, $Q\Lambda Q^T$?

3. (20pts) Section 6.4, Problem 31

   Suppose $A^T = -A$ (real antisymmetric matrix). Explain these facts about A:

   (a) $\mathbf{x}^T A\mathbf{x} = 0$ for every real vector $\mathbf{x}$.

   (b) The eigenvalues of A are pure imaginary.

   (c) The determinant of A is positive or zero (not negative).

   For (a), multiply out an example $\mathbf{x}^T A\mathbf{x}$ and watch terms cancel. Or reverse $\mathbf{x}^T(A\mathbf{x})$ to $(A\mathbf{x})^T\mathbf{x}$. For (b), $A\mathbf{z} = \lambda\mathbf{z}$ leads to $\overline{\mathbf{z}}^T A\mathbf{z} = \lambda\overline{\mathbf{z}}^T\mathbf{z} = \lambda\|\mathbf{z}\|^2$. Part (a) shows that $\overline{\mathbf{z}}^T A\mathbf{z} = (\mathbf{x} - i\mathbf{y})^T A(\mathbf{x} + i\mathbf{y})$ has two real part. Then (b) helps with (c).

4. (10pts) For what numbers a and b are A and B positive definite?

   $$A = \begin{bmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & b & 4 \\ 3 & 4 & 5 \end{bmatrix}.$$

5. (20pts) Section 6.5, Problem 20

   Give a quick reason why each of these statements is true:

   (a) Every positive definite matrix is invertible.

(b) The only positive definite projection matrix is $P=I$.

(c) A diagonal matrix with positive diagonal entries is positive definite.

(d) A symmetric matrix with a positive determinant might not be positive definite.

6. (10pts) Section 6.5, Problem 31

   Which values of c give a bowl and which c give a saddle point for the graph of $z = 4x^2 + 12xy + cy^2$? Describe this graph at the borderline value of c.