Linear Algebra

Problem Set 2 **2010**

Due Wednesday, 10 March 2010 at 10:00 AM in EE102. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (15pts) Find the inverses (in any legal way) of

$$A=\begin{bmatrix}0&0&0&1\\0&0&2&0\\0&3&0&0\\4&0&0&0\end{bmatrix} \quad \text{and} \quad B=\begin{bmatrix}1&2&0&0\\3&7&0&0\\0&0&5&3\\0&0&8&5\end{bmatrix}.$$

   (Actually, you don't need to use Gauss-Jordan method. Find the inverses by inspection.)

2. (10pts) Without finding A^{-1}, use Gauss-Jordan method to solve the following equation:

$$AX=\begin{bmatrix}1&3&1\\0&2&1\\4&1&1\end{bmatrix}X=\begin{bmatrix}1&9&2\\2&6&1\\-2&13&5\end{bmatrix}.$$

3. (15pts) Suppose that A, B, and $A-B$ are invertible $n\times n$ matrices. By direct multiplication, show that

$$(A-B)^{-1}=A^{-1}+A^{-1}(B^{-1}-A^{-1})^{-1}A^{-1}.$$

4. (20pts) Suppose the matrix $\begin{bmatrix}1&3&4&a&b\\2&1&5&3&0\\0&4&2&-4&c\end{bmatrix}$ can be transformed to

   $\begin{bmatrix}1&0&0&-6&f\\0&1&0&d&-6\\0&0&1&e&-4\end{bmatrix}$ with a series of row operations. Find a, b, c, d, e, and f.

   (Actually, you don't need to use Gaussian elimination. The 3 by 3 identity submatrix in the second matrix provides a clue to solve the problem in an easy way.)

5. (20pts) Suppose $B=(I-A)(I+A)^{-1}$ and $A=\begin{bmatrix}1&0&0&0\\2&1&0&0\\0&4&1&0\\0&0&6&1\end{bmatrix}$. Find $(I+B)^{-1}$.

(Please don't do a lot of calculations! Try removing $(I + A)^{-1}$ from the equation by multiplying some matrix to the right.)

6. (20pts) Suppose that *A*, *B*, and *A*+*B* are invertible. Show that $I + BA^{-1}$ is invertible. This can be done by finding out its inverse directly, that is, find *X* such that $(I + BA^{-1})X = I$. Use this result to prove that

$$A(A+B)^{-1}B = B(A+B)^{-1}A = (A^{-1} + B^{-1})^{-1}.$$

(One way to find the inverse of $A^{-1} + B^{-1}$ is to express this matrix sum as a sequence of matrix multiplications, say *XYZ*, and then use the rule: $(XYZ)^{-1} = Z^{-1}Y^{-1}X^{-1}$.)