Linear Algebra
Problem Set 3 **2010**

Due Wednesday, 17 March 2010 at 10:00 AM in EE102. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (20pts)
   (a) An n by n matrix A is called *idempotent* if $A^2 = A$, where $A^2 = AA$. If A is idempotent, find the inverse of $I - cA$ (if possible) for some scalar c. (Will the inverse of $I - cA$ look like $I - dA$?)
   (b) Let E be the n by n matrix each of whose entries is 1. What is the inverse of $I - E$? (What is the relationship between E and E^2?)
2. (20pts) Let B be a skew-symmetric matrix, $B^T = -B$. If $A = (I + B)(I - B)^{-1}$, prove that $A^{-1} = A^T$.
3. (15pts) Let
$$A = \begin{bmatrix} 1 & 2 & 4 & 17 \\ 3 & 6 & -12 & 3 \\ 2 & 3 & -3 & 2 \\ 0 & 2 & -2 & 6 \end{bmatrix}.$$
   Find the permutation matrix P as well as the LU factors such that $PA=LU$.
4. (15pts) If A is a matrix that contains only integer entries and all of its pivots are 1, explain why A^{-1} must also be an integer matrix. (Think of LU factorization. What is the inverse of L?) Use this fact to create a 3 by 3 invertible matrix $A, A \neq I$, satisfying the above requirements. Show A and its inverse.
5. (15pts) If A is symmetric and possesses an LDU factorization, explain why it must be given by $A = LDL^T$.
6. (15pts) Determine the inverse of the block matrix $\begin{bmatrix} A & 0 \\ B & C \end{bmatrix}$, where A is m by m, and C is n by n. What conditions must be satisfied so that the block matrix is invertible?