Linear Algebra

Problem Set 4 **2010**

Due Wednesday, 24 March 2010 at 10:00 AM in EE102. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (18pts) Section 3.1, Problem 10

   Which of the following subsets of $\mathbf{R}^3$ are actually subspaces?

   (a) The plane of vectors (b_1, b_2, b_3) with b_1=b_2.

   (b) The plane of vectors with b_1=1.

   (c) The vectors with $b_1b_2b_3$=0.

   (d) All linear combinations of $\mathbf{v}=(1,4,0)$ and $\mathbf{w}=(2,2,2)$.

   (e) All vectors that satisfy $b_1+b_2+b_3=0$.

   (f) All vectors with $b_1 \le b_2 \le b_3$.

2. (12pts) Section 3.1, Problem 27

   True or false (with a counterexample if false):

   (a) The vectors **b** are not in the column space $C(A)$ form a subspace.

   (b) If $C(A)$ contains only the zero vector, then A is the zero matrix.

   (c) The column space of $2A$ equals the column space of A.

   (d) The column space of $A-I$ equals the column space of A (test this).

3. (18pts) Section 3.1, Problem 30

   Suppose $\mathcal{S}$ and $\mathcal{T}$ are two subspaces of a vector space $\mathcal{V}$.

   (a) Definition: The **sum** $\mathcal{S}$+$\mathcal{T}$ contains all sums **s**+**t** of a vector **s** in $\mathcal{S}$ and a vector **t** in $\mathcal{T}$. Show that $\mathcal{S}$+$\mathcal{T}$ satisfies the requirements (addition and scalar multiplication) for a vector space (subspace).

   (b) If $\mathcal{S}$ and $\mathcal{T}$ are lines in $\mathbf{R}^m$, what is the difference between $\mathcal{S}$+$\mathcal{T}$ and $\mathcal{S}\cup\mathcal{T}$? That union contains all vectors from $\mathcal{S}$ or $\mathcal{T}$ or both. Explain this statement: The span of $\mathcal{S}\cup\mathcal{T}$ is $\mathcal{S}$+$\mathcal{T}$. (The span of $\mathcal{S}\cup\mathcal{T}$ means that the subspace is constructed by spanning any vectors in $\mathcal{S}\cup\mathcal{T}$.)

4. (12pts)

   (a) How is the column space $C(X)$ related to the spaces $C(A)$ and $C(B)$, if

   $$X=\begin{bmatrix} A & B \end{bmatrix}.$$

   (b) How is the nullspace $N(X)$ related to the spaces $N(A)$ and $N(B)$, if $X=\begin{bmatrix} A \\ B \end{bmatrix}$.

5. (10pts) Find the nullspace matrix N associated with the following reduced row echelon form R:

$$R=\begin{bmatrix}1 & -3 & 0 & 5 & 0 & 0 & 7\\ 0 & 0 & 1 & -4 & 0 & 0 & 3\\ 0 & 0 & 0 & 0 & 1 & 0 & 2\\ 0 & 0 & 0 & 0 & 0 & 1 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}.$$

Note that the columns of N span the nullspace of R, that is, $N(R)=C(N)$.

6. (30pts) Section 3.3, Problem 27

   Suppose R is m by n of rank r, with pivot columns first:

   $$R=\begin{bmatrix}I & F\\ 0 & 0\end{bmatrix}.$$

   (a) What are the shapes of those four blocks?

   (b) Find a *right-inverse* B with $RB=I$ if $r=m$.

   (c) Find a *left-inverse* C with $CR=I$ if $r=n$.

   (d) What is the reduced row echelon form of R^T (with shapes)?

   (e) What is the reduced row echelon form of R^TR (with shapes)?

   (f) Prove that R^TR has the same nullspace as R.