Linear Algebra

Problem Set 5 **2010**

Due Wednesday, 31 March 2010 at 10:00 AM in EE102. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (15pts) Section 3.4, Problem 34
   Suppose you know that the 3 by 4 matrix A has the vector $\mathbf{s} = (2,3,1,0)$ as the only special solution to $A\mathbf{x} = \mathbf{0}$.
   (a) What is the rank of A and the complete solution to $A\mathbf{x} = \mathbf{0}$?
   (b) What is the exact row reduced row echelon form R of A?
   (c) How do you know that $A\mathbf{x} = \mathbf{b}$ can be solved for all $\mathbf{b}$?
2. (20pts) Section 3.5, Problem 16
   Find a basis for each of these subspaces of $\mathbf{R}^4$:
   (a) All vectors whose components are equal.
   (b) All vectors whose components add to zero.
   (c) All vectors that are perpendicular to $(1,1,0,0)$ and $(1,0,1,1)$.
   (d) All vectors of the form (a,b,c,d), where $c = a+b$ and $d = c-b$.
   (For each problem, try to translate the requirement into the column space or nullspace of a matrix. That would make life easier.)
3. (15pts) Section 3.5, Problem 41
   Write the 3 by 3 identity matrix as a linear combination of the other five permutation matrices! Then show that those five matrices are linearly independent. (Assume a combination gives $c_1P_1 + \cdots + c_5P_5 =$ zero matrix, and check entries to prove c_i is zero.) Show the five permutations are a basis for the subspace of 3 by 3 matrices with row and column sums all equal.
4. (20pts) Construct a matrix with the required property or explain why this is impossible:

   (a) Column space contains $\begin{bmatrix}1\\1\\0\end{bmatrix},\begin{bmatrix}0\\0\\2\end{bmatrix}$, row space contains $\begin{bmatrix}1\\2\end{bmatrix},\begin{bmatrix}2\\3\end{bmatrix}$.

   (b) Column space has basis $\begin{bmatrix}1\\1\\3\end{bmatrix}$, nullspace has basis $\begin{bmatrix}1\\0\\1\end{bmatrix}$.

   (c) Dimension of nullspace = 1 + dimension of left nullspace.

(d) Left nullspace contains $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, row space contains $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(e) Row space = column space, nullspace $\neq$ left nullspace.

(The rank theorem is your friend.)

5. (14pts) Suppose the general solution to the equation

$$A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix} \quad \text{is} \quad \mathbf{x} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}.$$

(a) Find the reduced row echelon form of A.

(b) Find a basis for the column space of A.

(c) Find a basis for the nullspace of A.

6. (16pts) The reduced row echelon form of $\begin{bmatrix} A & I_3 \end{bmatrix}$ is as follows:

$$\begin{bmatrix} 1 & 3 & 0 & -2 & -1 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 & 3 & 2 & -3 & -2 \\ 0 & 0 & 0 & 0 & 0 & 3 & -6 & -8 \end{bmatrix}$$

Find bases for the four fundamental subspaces of A. (Note that A is unknown, can you recover it?)