Linear Algebra
Problem Set 6 **2010**

Due Wednesday, 21 April 2010 at 10:00 AM in EE102. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (20pts) Find the range and kernel (like the column space and nullspace) of T:

   (a) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 \\ 0 \end{bmatrix}$ (b) $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 + x_3 \end{bmatrix}$

   (c) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 \\ x_1 \end{bmatrix}$ (d) $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

   Which transformations are one-to-one? Which are onto?
2. (15pts) Suppose $T : \mathbf{R}^n \to \mathbf{R}^m$ is a linear transformation.
   (a) If T maps $\mathbf{R}^n$ onto $\mathbf{R}^m$, what is the relationship between m and n?
   (b) If T is onto-to-one, what can you say about m and n?
   (c) If T is onto-to-one and maps onto $\mathbf{R}^m$, what can you say about m and n?
3. (15pts) Section 7.1, Problem 31
   Why does every linear transformation T from $\mathbf{R}^2$ to $\mathbf{R}^3$ takes squares to parallelograms? Rectangles also go to parallelograms (squashed if T is not invertible).
4. (10pts) Use coordinate vectors to verify that the polynomials $1-t+2t^2$, $2-t+5t^2$, and $-1+4t+2t^2$ are linearly independent in $\mathbf{P}^2$.
5. (25pts) Suppose T is a linear transformation from $\mathbf{R}^3$ to $\mathbf{R}^3$ and

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}, \quad T\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}.$$

   (a) Find the matrix representation of T with respect to the standard basis.
   (b) Find the matrix representation of T with respect to the basis

$$\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right\}.$$

(c) Find the matrix representation of T with respect to the basis

$$\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}2\\3\\2\end{bmatrix}, \begin{bmatrix}1\\0\\-1\end{bmatrix}\right\}.$$

6. (15pts) Section 7.2, Problem 10
Suppose $T(\mathbf{v}_1) = \mathbf{w}_1 + \mathbf{w}_2 + \mathbf{w}_3$ and $T(\mathbf{v}_2) = \mathbf{w}_2 + \mathbf{w}_3$ and $T(\mathbf{v}_3) = \mathbf{w}_3$. Find the matrix A for T using these basis vectors. What input vector $\mathbf{v}$ gives $T(\mathbf{v}) = \mathbf{w}_1$?