Linear Algebra

Problem Set 8 **2010**

Due Wednesday, 5 May 2010 at 10:00 AM in EE102. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (15pts) Section 4.3, Problem 1

   With $b = 0, 8, 8, 20$ at $t = 0, 1, 3, 4,$ set up and solve the normal equations $A^T A\hat{\mathbf{x}} = A^T\mathbf{b}$. For the best line in Figure 4.9a, find its four heights p_i and four errors e_i. What is the minimum value $E = e_1^2 + e_2^2 + e_3^2 + e_4^2$?

2. (15pts) Section 4.3, Problem 9

   For the closest parabola $b = C + Dt + Et^2$ to the same four points, write down the unsolvable equations $A\mathbf{x} = \mathbf{b}$ in three unknowns $\mathbf{x} = (C, D, E)$. Set up the three normal equations $A^T A\hat{\mathbf{x}} = A^T\mathbf{b}$ (solution not required). In Figure 4.9a you are now fitting a parabola to 4 points—what is happening in Figure 4.9b?

3. (20pts) Express the Gram-Schmidt orthogonalization of $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$ as $A = QR$. With the same matrix A, and with $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, use $A = QR$ to solve the least squares problem $A\mathbf{x} = \mathbf{b}$.

4. (20pts) Let $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 3 \end{bmatrix}$. Find the projection matrices onto the column space, row space, nullspace, and left nullspace of A, respectively.

5. (15pts) Section 4.4, Problem 24

   (a) Find a basis for the subspace S in $\mathbf{R}^4$ spanned by all solutions of
   $$x_1 + x_2 + x_3 - x_4 = 0.$$

   (b) Find a basis for the orthogonal complement $S^{\perp}$.

   (c) Find $\mathbf{b}_1$ in S and $\mathbf{b}_2$ in $S^{\perp}$ so that $\mathbf{b}_1 + \mathbf{b}_2 = \mathbf{b} = (1,1,1,1)$.

6. (15pts) Suppose m by n matrix Q contains orthonormal columns. It is known that $P = QQ^T$ is the projection onto the column space of Q. Now add another column **a** to produce $A = [Q \quad \mathbf{a}]$. What is the new orthonormal vector **q** from Gram-Schmidt?