Linear Algebra

Problem Set 9 **2010**

Due Wednesday, 12 May 2010 at 10:00 AM in EE102. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (20pts) Section 5.1, Problem 15

   Use row operations to simplify and compute these determinants:

$$\det\begin{bmatrix} 101 & 201 & 301 \\ 102 & 202 & 302 \\ 103 & 203 & 303 \end{bmatrix} \text{ and } \det\begin{bmatrix} 1 & t & t^2 \\ t & 1 & t \\ t^2 & t & 1 \end{bmatrix}.$$

2. (15pts) Section 5.2, Problem 20

   Fing G_2 and G_3 and then by row operations G_4. Can you predict G_n?

$$G_2 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \quad G_3 = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} \quad G_4 = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}.$$

3. (15pts) For each of the following matrices, find all possible values of determinant.
   (a) P is n by n, and P is a permutation matrix.
   (b) P is n by n, and P is a projection matrix.
   (c) A is n by n, n is odd, and A is skew-symmetric, i.e., $A^T = -A$.
4. (15pts) Section 5.2, Problem 25

   Block elimination subtracts CA^{-1} times the first row $\begin{bmatrix} A & B \end{bmatrix}$ from the second row $\begin{bmatrix} C & D \end{bmatrix}$. This leaves the *Schur complement* $D - CA^{-1}A$ in the corner:

$$\begin{bmatrix} I & 0 \\ -CA^{-1} & I \end{bmatrix}\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & D - CA^{-1}B \end{bmatrix}.$$

   Take determinants of these block matrices to prove correct rules if A^{-1} exists:

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = |A| \cdot |D - CA^{-1}B| = |AD - CB| \text{ provided } AC = CA.$$

5. (20pts) Use determinants to answer the following questions.
   (a) A box has edges from (0,0,0) to (3,1,1) and (1,3,1) and (1,1,3). Find its volume.
   (b) Find the area of the parallelogram with sides from (0,0,0) to (3,1,1) and (1,3,1).

(c) Find the area of the triangular with corners (3,1,1), (1,3,1) an (1,1,3).

6. (15pts) Section 5.3, Problem 39

   If you know all 16 cofactors of a 4 by 4 invertible matrix A, how would you find A?