Matrix Theory

Problem Set 1 **2010**

Due Thursday, 30 September 2010 at 12 noon in EE105. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print every group member's name and student ID on your homework.

1. (18pts) True or false. Give a specific example when false:
   (a) $\{(x, y) | x^2 + y^2 = 0,\ x, y \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^2$.

   (b) $\{(x, y) | x^2 + y^2 = 0,\ x, y \in \mathbb{C}\}$ is a subspace of $\mathbb{C}^2$.

   (c) $\{(x, y) | x^2 + y^2 = 1,\ x, y \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^2$.

   (d) $\{(x, y) | x - y = 1,\ x, y \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^2$.

   (e) $\{(x, y) | x = y,\ x, y \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^2$.

   Let $\mathbb{P}_n$ be the set of real polynomials of degree less then *n*.

   (f) $\{p(t) | p(1) = 0,\ p(t) \in \mathbb{P}_n\}$ is a subspace of $\mathbb{P}_n$.

   (g) $\{p(t) | p(t) \in \mathbb{P}_n \text{ has degree } 2\}$ is a subspace of $\mathbb{P}_n$.

   (h) $\{p(t) | p(0) = p(1),\ p(t) \in \mathbb{P}_n\}$ is a subspace of $\mathbb{P}_n$.

   (i) $\{p(t) | p(t) \geq 0,\ p(t) \in \mathbb{P}_n\}$ is a subspace of $\mathbb{P}_n$.

2. (16pts) Let W_1 and W_2 be subspaces of a vector space *V*. The *sum* of W_1 and W_2 is defined by $W_1 + W_2 = \{\mathbf{w}_1 + \mathbf{w}_2 | \mathbf{w}_1 \in W_1, \mathbf{w}_2 \in W_2\}$.
   (a) Prove that $W_1 + W_2$ and $W_1 \cap W_2$ are subspaces, and
   $$W_1 \cap W_2 \subseteq W_1 \cup W_2 \subseteq W_1 + W_2.$$
   (b) Suppose $V = \mathbb{R}^2$. Give an example to show that $W_1 \cup W_2$ is not a subspace of *V*.
   (c) When is $W_1 \cup W_2$ a subspace?

3. (20pts) Find the dimension of the subspace W of $\mathbb{R}^4$ given by

   (a) $W = \{(a,b,c,d) | a+b+c+d = 0\}$

   (b) $W = \{(a,b,c,d) | d = 0\}$

   (c) $W = \{(a,b,c,d) | c = a-b, d = a+b\}$

   (d) $W = \{(a,b,c,d) | a = b = c = d\}$

4. (16pts) Let T be a linear transformation on $\mathbb{R}^3$ defined by

$$T\begin{pmatrix} x \\ y \\ x \end{pmatrix} = \begin{pmatrix} x-3y \\ y+z \\ 0 \end{pmatrix}.$$

   Find the dimensions of Ran(T) and Ker(T), and find their bases. Can you find a linear transformation L on $\mathbb{R}^3$ such that dimRan(L)=2 and Ran(L)$\subseteq$Ker(L)? Why or why not?

5. (30pts) MATLAB Problem. An n by n 0-1 matrix has entries 0 or 1. Use the random number generator to produce random matrices. Answer the following questions.
   (a) What fraction of them are singular (not invertible)?
   (b) What are the frequencies of different determinants? Draw the histogram of your results.
   (c) What is the largest possible determinant?

   Start with n=2, and increase n gradually and see how your answers in (a), (b), (c) change with n. Write down your findings in this computer experiment.