Matrix Theory

Problem Set 6 **2010**

Due <u>Thursday, 25 November 2010</u> at 12 noon in EE105. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print every group member's name and student ID on your homework.

1. (15pts) Answer the following questions.
   (a) Show that $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ are similar.
   (b) Show that $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ are similar.
   (c) Show that $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ are not similar.
2. (10pts) For a scalar t, determine the matrix exponential e^{At}, where $A = \begin{bmatrix} -a & b \\ a & -b \end{bmatrix}$, with $a + b \neq 0$.
3. (20pts) If A is $n \times n$ and diagonalizable, $A = S\Lambda S^{-1}$, we define the following matrix function: $f(A) = Sf(\Lambda)S^{-1}$, where $f(\Lambda) = \begin{bmatrix} f(\lambda_1) & & \\ & \ddots & \\ & & f(\lambda_n) \end{bmatrix}$.
   (a) Determine $\cos A$ for $A = \begin{bmatrix} -\pi/2 & \pi/2 \\ \pi/2 & -\pi/2 \end{bmatrix}$.
   (b) Explain why $\cos^2 A + \sin^2 A = I$ for a diagonalizable matrix A.
4. (15pts) Answer the following questions.
   (a) Let A be an $n \times n$ nonsingular real matrix. Show that $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^T A^T A \mathbf{y}$ is an inner product for $\mathbb{R}^n$. This can be done by proving that the above definition of $\langle \mathbf{x}, \mathbf{y} \rangle$ satisfies the four axioms of inner product spaces.
   (b) Let A and B be $m \times n$ complex matrices. Show that $C(A) \perp C(B)$ if and only if $A^* B = 0$, where $A^* = \overline{A}^T$.
5. (15pts) If $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, determine the orthogonal projection matrices onto each of the four fundamental subspaces of A.

6. (25pts) MATLAB Problem. In this assignment, you are asked to use the Power method to compute the dominant (largest absolute) eigenvalue λ of A with unit eigenvector $\|\mathbf{x}\| = 1$.

(1) Input an initial guess $\mathbf{x}_0$ for $\mathbf{x}$.

(2) For $k = 0, 1, 2, \ldots$ until convergence of $\lambda(k)$:

$$\mathbf{y} = A\mathbf{x}_k$$

$$\mathbf{x}_{k+1} = \mathbf{y} / \|\mathbf{y}\|$$

$$\lambda(k+1) = \mathbf{x}_{k+1}^T A \mathbf{x}_{k+1.}$$

(a) Let $A = \begin{bmatrix} -8 & -5 & 8 \\ 6 & 3 & -8 \\ -3 & 1 & 9 \end{bmatrix}$. Find the dominant eigenvalue and corresponding eigenvector of A.

(b) Let $B = \begin{bmatrix} 1 & -3 & 9 \\ 0 & 2 & -4 \\ 0 & 0 & k \end{bmatrix}$. Find the dominant eigenvalue of B, starting with $k = 3$, and reducing k gradually till $k = 2$. Describe your findings in this experiment. Can you explain whatever results you got?