Matrix Theory

Problem Set 8 **2010**

Due Thursday, 16 December 2010 at 12 noon in EE105. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print every group member's name and student ID on your homework.

1. (20pts) Answer the following questions.
   (a) If A is similar to A^{-1}, must all the eigenvalues of A equal to 1 or -1? If your answer is yes, prove it; otherwise, give a counter example.
   (b) Are the following matrices similar? Justify your answer.

$$A=\begin{bmatrix}1&0&0\\1&1&0\\0&1&1\end{bmatrix},\quad B=\begin{bmatrix}1&2&0\\0&1&2\\0&0&1\end{bmatrix},\quad C=\begin{bmatrix}1&2&0\\2&1&0\\0&0&1\end{bmatrix}.$$

2. (10%) Find the Jordan canonical form J of the matrix

$$A=\begin{bmatrix}3&-7&-2\\1&-2&-1\\-3&9&4\end{bmatrix}.$$

   Also, find an invertible matrix M such that $A=MJM^{-1}$.

3. (35pts) Answer the following questions.
   (a) Find the Jordan form of the following matrix

$$\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\cdots&1\\1&0&0&\cdots&0\end{bmatrix}.$$

   (b) If J is a Jordan block with eigenvalue λ, find the Jordan forms of J^{-1} (if it exists) and J^2.
   (c) Explain why every nonsingular complex matrix A has a square root, i.e., there exists B such that $B^2=A$.

4. (15pts) Note that the following matrices are m by m. With the convention that $\binom{k}{j}=0$ for $j>k$, explain that

$$\begin{bmatrix} \lambda & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & \lambda \end{bmatrix}^k = \begin{bmatrix} \lambda^k & \binom{k}{1}\lambda^{k-1} & \binom{k}{2}\lambda^{k-2} & \cdots & \binom{k}{m-1}\lambda^{k-m+1} \\ & \lambda^k & \binom{k}{1}\lambda^{k-1} & \ddots & \vdots \\ & & \ddots & \ddots & \binom{k}{2}\lambda^{k-2} \\ & & & \lambda^k & \binom{k}{1}\lambda^{k-1} \\ & & & & \lambda^k \end{bmatrix}.$$

5. (20pts) Answer the following questions.
   (a) Show that if $AB = BA$, then $e^{A+B} = e^A e^B$.
   (b) Give an example to illustrate that $e^{A+B} \neq e^A e^B$, in general.