Linear Algebra

Problem Set 1 **Spring 2012**

Due Tuesday, 6 March 2012 at 10:00 AM in EE208. This problem set covers Lecture 1-3. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (15pts) Let $A = \begin{bmatrix} 4 & -1 & 3 \\ -2 & 2 & 0 \\ 1 & -2 & -1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Show that the equation $A\mathbf{x} = \mathbf{b}$ does not have a solution for all possible **b**, and describe the set of all **b** for which $A\mathbf{x} = \mathbf{b}$ does have a solution.
2. (20pts) Consider the system of linear equations:
$$\begin{aligned} x + (a-1)y + (a-2)z &= 2 \\ ax + 2(a-1)y + (a-2)z &= a+1 \\ a(a-2)x + az &= -2a. \end{aligned}$$
Find the real values of a such that this system of equations satisfies the following conditions.
   (a) The system has no solution.
   (b) The system has a unique solution.
   (c) The system has infinite many solutions.
3. (20pts) Let
$$A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}.$$
   (a) Find elimination matrices E_{21} then E_{32} then E_{43} to change A into an echelon form U, i.e., $E_{43}E_{32}E_{21}A = U$.
   (b) Apply those three steps to the identity matrix I, to multiply $E_{43}E_{32}E_{21}$.
4. (15pts) The three types of elementary row operations are not independent: the row exchange operation can be accomplished by a sequence of the other two types of row operations, namely, replacing (i.e., subtracting $k \times$ row$i$ from row$j$) and scaling (i.e., multiplying row$i$ by k, $k \neq 0$). Apply a sequence of replacing and scaling operations to the identify matrix I to obtain the permutation matrix:
$$P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

5. (15pts) Without using Gaussian elimination, solve X by inspection. Identify the size of X first.

(a) $$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix} X = \begin{bmatrix} 1 & 0 & 4 & 4 & 5 \\ 2 & 2 & 4 & 5 & 5 \\ 3 & 0 & 4 & 6 & 6 \\ 4 & 0 & 4 & 7 & 7 \end{bmatrix}$$

(b) $$X \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

6. (15pts) Consider the following three systems where the coefficients are the same for each system, but the right-hand sides are different:

$$\begin{aligned} 3x + 2y - 2z &= 1 \,|\, 0 \,|\, 0 \\ -x - y + z &= 0 \,|\, 1 \,|\, 0 \\ x + 3y - 2z &= 0 \,|\, 0 \,|\, 1. \end{aligned}$$

Solve all three systems at one time by performing Gaussian elimination on an augmented matrix of the form $\left[A \middle| \mathbf{b}_1 \middle| \mathbf{b}_2 \middle| \mathbf{b}_3\right]$. What is the 3 by 3 matrix X such that $XA=I$?