Linear Algebra
Problem Set 1 **Spring 2013**

Due Tuesday, 5 March 2013 at 12:00 PM in EE208. This problem set covers Lecture 1-4. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (10pts) Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 0 & 0 \\ 2 & -4 & 6 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Show that the equation $A\mathbf{x} = \mathbf{b}$ does not have a solution for all possible **b**, and describe the set of all **b** for which $A\mathbf{x} = \mathbf{b}$ does have a solution.
2. (10pts) Write down the 3 by 3 matrices that produce these elimination steps:
   (a) E_{21}: subtract 4 times row1 from row2.
   (b) E_{32}: subtract -7 times row2 from row3.
   (c) P: exchange row1 and row2 , then row2 and row3.
3. (10pts) Let
   $$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}.$$
   Find elimination matrices E_{21} then E_{32} then E_{43} to change A into an echelon form U, i.e., $E_{43}E_{32}E_{21}A = U$.
4. (10pts) Consider the following three systems where the coefficients are the same for each system, but the right-hand sides are different:
   $$x - y = 1|0|0$$
   $$2x - y + z = 0|1|0$$
   $$2x - 2y + z = 0|0|1$$
   Solve all three systems at one time by performing Gaussian elimination on an augmented matrix of the form $\left[A|\mathbf{b}_1|\mathbf{b}_2|\mathbf{b}_3\right]$. What is the 3 by 3 matrix X such that $XA=I$?
5. (15pts) The three types of elementary row operations are not independent: the row exchange operation can be accomplished by a sequence of the other two types of row operations, namely, replacing (i.e., subtracting $k \times$ row*i* from row*j*) and scaling (i.e., multiplying row*i* by k, $k \neq 0$). Apply a sequence of replacing and scaling operations to the 3 by 3 identify matrix I to obtain the permutation matrix:

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

6. (15pts) True or false. Give a specific example when false:
   (a) If columns 2 and 4 of B are the same, so are columns 2 and 4 of AB.
   (b) If rows 2 and 4 of B are the same, so are rows 2 and 4 of AB.
   (c) If rows 2 and 4 of A are the same, so are rows 2 and 4 of ABC.
   (d) $(AB)^2 = A^2B^2$.
   (e) $(A+B)^2 = A^2+2AB+B^2$.
7. (15pts) Without using Gaussian elimination, solve X by inspection. Identify the size of X first.
   (a) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} 4 & 0 & 3 & 5 & 3 \\ 3 & 0 & 3 & 4 & 3 \\ 2 & 1 & 3 & 3 & 6 \\ 1 & 0 & 3 & 2 & 6 \end{bmatrix}$
   (b) $X \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$
8. (15pts) Section 2.4, Problem 36 (Show every step of your proof.)
   To prove that $(AB)C = A(BC)$, use the column vectors $\mathbf{b}_1,\ldots,\mathbf{b}_n$ of B. First suppose that C has only one column $\mathbf{c}$ with entries $c_1,\ldots,c_n$:
   AB has columns $A\mathbf{b}_1,\ldots,A\mathbf{b}_n$ and then $(AB)\mathbf{c}$ equals $c_1A\mathbf{b}_1+\ldots+c_nA\mathbf{b}_n$.
   $B\mathbf{c}$ has one column $c_1\mathbf{b}_1+\ldots+c_n\mathbf{b}_n$ and then $A(B\mathbf{c})$ equals $A(c_1\mathbf{b}_1+\ldots+c_n\mathbf{b}_n)$.
   Linearity gives equality of those two sums. This proves $(AB)\mathbf{c} = A(B\mathbf{c})$. The same is true for all other _____of C. Therefore $(AB)C = A(BC)$. Apply to inverses:
   If $BA=I$ and $AC=I$, prove that the left-inverse B equals the right-inverse C.