Linear Algebra
Problem Set 1 **Spring 2016**

Due Thursday, 3 March 2016 at 4:20 PM in EE105. This problem set covers Lectures 1-4. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (15pts) Consider the linear system

$$x+y-z=2$$
$$x+2y+z=3$$
$$x+y+(k^2-5)z=k.$$

   (a) For which value(s) of k does this system have a unique solution?
   (b) For which value(s) of k does the system have infinitely many solutions?
   (c) For which value(s) of k is the system inconsistent? A system is inconsistent if there is no solution.

2. (10pts) Find a polynomial of degree ≤ 2 (a polynomial of the form $f(t)=a+bt+ct^2$) whose graph goes through the points $(1, p_1)$, $(2, p_2)$, $(3, p_3)$, where p_1, p_2, p_3 are arbitrary constants. Does such a polynomial exist for all values of p_1, p_2, p_3?

3. (10pts) Is there a sequence of elementary row operations that transform

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \text{ into } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}?$$

   Explain.

4. (10pts) For which values of the constants a and b is the vector $\begin{bmatrix} 3 \\ a \\ b \end{bmatrix}$ a linear combination of $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$?

5. (10pts) Boris has 32 bills in his wallet, in the denominations of US$1, 5, and 10, worth $100 in total. How many does Boris have of each denomination?

6. (15pts) The three types of elementary row operations are *not independent*: the row exchange operation can be accomplished by a sequence of the other two types of

row operations, namely, replacing (i.e., subtracting $k \times$ row$i$ from row$j$) and scaling (i.e., multiplying row$i$ by k, $k \neq 0$). Apply a sequence of replacing and scaling operations to the 3 by 3 identity matrix I to obtain the permutation matrix:

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

7. (20pts) True or false. Justify your answer.
   (a) If columns 1 and 2 of A are the same, so are columns 1 and 2 of AB.
   (b) If rows 2 and 3 of A are the same, so are rows 2 and 3 of ABC.
   (c) There exists a 2 by 2 matrix A such that $A\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $A\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.
   (d) Let A be a 3 by 3 matrix. If $A\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has a unique solution, then

   $A\mathbf{x} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ has a unique solution.

   (e) Let A be a 3 by 3 matrix. If $A\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has infinitely many solutions, then

   $A\mathbf{x} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ has infinitely many solutions.

8. (10pts) Find X by inspection. Identify the size of X first.
   (a) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 & 3 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$
   (b) $X \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}.$