Linear Algebra
Problem Set 10 **2010**

Due Wednesday, 26 May 2010 at 10:00 AM in EE102. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (15pts) Find the rank and the eigenvalues of A and B:

$$A=\begin{bmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix} \text{ and } B=\begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&1&0\\0&1&0&1\end{bmatrix}.$$

2. (20pts) Show that the eigenvalues of A equal the eigenvalues of A^T. (Hint: use characteristic polynomial to prove it.) Also show by an example that the eigenvectors of A and A^T are not the same.
3. (15pts) A 3 by 3 matrix A is known to have eigenvalues of $-1,0,1$. This information is enough to find three of these (give the answers where possible):
   (a) The rank of A
   (b) The determinant of A^TA
   (c) The eigenvalues of A^TA
   (d) The eigenvalues of $(A^2+I)^{-1}$.
4. (15pts) Section 6.1, Problem 37
   (a) Find the eigenvalues and eigenvectors of A. They depend on c:

$$A=\begin{bmatrix}0.4&1-c\\0.6&c\end{bmatrix}.$$

   (b) Show that A has just one line of eigenvectors when c = 1.6.
   (c) This is a Markov matrix when c = 0.8. Then A^n will approach what matrix A^∞ ?
5. (20pts) Section 6.2, Problem 9
   Suppose G_{k+2} is the *average* of two previous numbers G_{k+1} and G_k:

$$\begin{aligned}G_{k+2}&=\frac{1}{2}(G_{k+1}+G_k)\\G_{k+1}&=G_{k+1}\end{aligned} \quad \text{is} \quad \begin{bmatrix}G_{k+2}\\G_{k+1}\end{bmatrix}=\left[A\right]\begin{bmatrix}G_{k+1}\\G_k\end{bmatrix}.$$

   (a) Find the eigenvalues and eigenvectors of A.
   (b) Find the limit as $n\to\infty$ of the matrix $A^n=S\Lambda^nS^{-1}$.
   (c) If G_0 = 0 and G_1 = 1 show that the Gibonacci numbers approach 2/3.
6. (15pts) Section 6.2, Problem 32

   Substitute $A=S\Lambda S^{-1}$ into the product $(A-\lambda_1 I)(A-\lambda_2 I)\cdots(A-\lambda_n I)$ and explain why this produces the zero matrix. We are substituting the matrix A for the number λ in the polynomial $p(\lambda)=\det(A-\lambda I)$. The Cayley-Hamilton Theorem says that this product is always $p(A) =$ zero matrix, even if A is not diagonalizable.