Linear Algebra

Problem Set 10 **Spring 2012**

Due Thursday, 31 May 2012 at 4:30 PM in EE208. This problem set covers Lecture 32-35. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (15pts) Let $A=\begin{bmatrix}2&2&2\\2&2&2\\2&2&2\end{bmatrix}$.
   (a) Find the eigenvalues of A.
   (b) Find the eigenvectors of A.
2. (15pts) A 3 by 3 matrix A is known to have eigenvalues of $1,2,3$. Use this information to find the values of the following items (give the answers where possible):
   (a) The rank of A
   (b) The determinant of A^TA
   (c) The eigenvalues of A^TA
   (d) The trace of $A+2I$
   (e) The eigenvalues of $(A^2+I)^{-1}$.
3. (15pts) Let $A=\begin{bmatrix}3&2\\-5&-3\end{bmatrix}$ and $B=\begin{bmatrix}5&7\\-3&-4\end{bmatrix}$.
   (a) Show that $A^{1024}=I$.
   (b) Show that $B^{1024}=-B$.
4. (20pts)
   (a) The block B has eigenvalues 1, 2 and C has eigenvalues 3, 4 and D has eigenvalues 5, 7. Find the eigenvalues of the 4 by 4 matrix A:

$$A=\begin{bmatrix}B&C\\0&D\end{bmatrix}=\begin{bmatrix}0&1&3&0\\-2&3&0&4\\0&0&6&1\\0&0&1&6\end{bmatrix}.$$

   (b) Find the determinants of $A=\begin{bmatrix}0&1&1&1\\1&0&1&1\\1&1&0&1\\1&1&1&0\end{bmatrix}$. (Try to find the eigenvalues of A and then multiply them together.)

5. (15pts)
   If $a_{k+2} = 3a_{k+1} - 2a_k$, and $a_0 = 2,\ a_1 = 3,$ find the formula for a_k, where k is a nonnegative integer.
6. (20pts)
   (a) Substitute $A = S\Lambda S^{-1}$ into the product $(A-\lambda_1 I)(A-\lambda_2 I)\cdots(A-\lambda_n I)$ and

   explain why this produces the zero matrix. We are substituting the matrix A for the number λ in the polynomial $p(\lambda) = \det(A-\lambda I)$. The Cayley-Hamilton Theorem says that this product is always $p(A) =$ zero matrix, even if A is not diagonalizable.

   (b) Use the Cayley-Hamilton theorem to $A^3 + 3A^2 - 13A - 17I$, where

$$A = \begin{bmatrix} -3 & 2 & 0 \\ 2 & -3 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$