Linear Algebra

Problem Set 10 **Spring 2013**

Due Tuesday, 11 June 2013 at 12:00 PM in EE208. This problem set covers Lecture 38-41. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (20pts) Section 6.6, Problem 2 and 3

   Show that A and B are similar by finding M so that $B = M^{-1}AM$.

   (a) $A = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

   (d) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$.

2. (10pts) Section 6.6, Problem 6

   There are sixteen 2 by 2 matrices whose entries are 0's and 1's. Similar matrices go into the same family. How many families? How many matrices (total 16) in each family?

3. (15pts) Section 6.6, Problem 17, 18, 20

   True or false, with a good reason.

   (a) A symmetric matrix can't be similar to a nonsymmetric matrix.

   (b) An invertible matrix can't be similar to a singular matrix.

   (c) A can't be similar to $-A$ unless A=0.

   (d) A can't be similar to $A+I$.

   (e) If B is invertible, then AB is similar to BA.

   (f) If A is similar to B, then A^2 is similar to B^2.

   (g) A^2 and B^2 can be similar when A and B are not similar (try $\lambda = 0, 0$).

4. (15pts) Section 6.4, Problem 11, 12

   (a) Write A and B in the form of $\lambda_1 \mathbf{q}_1 \mathbf{q}_1^T + \lambda_2 \mathbf{q}_2 \mathbf{q}_2^T$ of the spectral theorem

   $Q\Lambda Q^T$: $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 9 & 12 \\ 12 & 16 \end{bmatrix}$ (keep $\|\mathbf{q}_1\| = \|\mathbf{q}_2\| = 1$)

   (b) Every 2 by 2 symmetric matrix is $\lambda_1 \mathbf{q}_1 \mathbf{q}_1^T + \lambda_2 \mathbf{q}_2 \mathbf{q}_2^T = \lambda_1 P_1 + \lambda_2 P_2$. Explain

   $P_1 + P_2 = \mathbf{q}_1 \mathbf{q}_1^T + \mathbf{q}_2 \mathbf{q}_2^T = I$ from columns time rows of Q. Why $P_1 P_2 = 0$?

5. (15pts) True or false, with a reason or counterexample if false.
   (a) A matrix with real eigenvalues and eigenvectors is symmetric.
   (b) A matrix with real eigenvalues and orthogonal eigenvectors is symmetric.
   (c) The inverse of a symmetric matrix is symmetric.
   (d) The eigenvector matrix S of a symmetric matrix is symmetric.
6. (10pts) Section 6.5, Problem 12
   For what numbers c and d are A and B positive definite? Test the 3 determinants:

$$A = \begin{bmatrix} c & 1 & 1 \\ 1 & c & 1 \\ 1 & 1 & c \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & d & 4 \\ 3 & 4 & 5 \end{bmatrix}.$$

7. (15pts) Section 6.5, Problem 20
   Give a quick reason why each of these statements is true:
   (a) Every positive definite matrix is invertible.
   (b) Every positive definite matrix has positive diagonal entries.
   (c) The only positive definite projection matrix is $P=I$.
   (d) A diagonal matrix with positive diagonal entries is positive definite.
   (e) A symmetric matrix with a positive determinant might not be positive definite.