Linear Algebra

Problem Set 11 **Spring 2012**

Due Tuesday, 12 June 2012 at 12:00 PM in EE208. This problem set covers Lecture 36-39. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (15pts) Section 6.3, Problem 3

   Solve $\frac{d\mathbf{u}}{dt} = \begin{bmatrix} -2 & 3 \\ 2 & -3 \end{bmatrix}\mathbf{u}$ with $\mathbf{u}(0) = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$. What is $\mathbf{u}(\infty)$?

2. (20pts) Section 6.3, Problem 31

   The cosine of a matrix is defined like e^A, by copying the series for cos *t*:

   $\cos t = 1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \cdots \quad \cos A = I - \frac{A^2}{2!} + \frac{A^4}{4!} - \cdots$

   (a) If $A\mathbf{x} = \lambda\mathbf{x}$, multiply each term times **x** to find the eigenvalue of cos *A*.

   (b) Find the eigenvalues of $A = \begin{bmatrix} \pi & \pi \\ \pi & \pi \end{bmatrix}$ with eigenvectors (1,1) and (1,-1).

   From the eigenvalues and eigenvectors of cos *A*, find that matrix $C = \cos A$.

3. (15pts) Find an invertible matrix *S* and a matrix *C* of the form $C = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ such that the given matrix has the form $A = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix} = SCS^{-1}$.

4. (15pts) Plot the trajectory of the system $\mathbf{u}_{k+1} = A\mathbf{u}_k$, where $A = \begin{bmatrix} 0.8 & -0.5 \\ -0.1 & 1.0 \end{bmatrix}$, with initial condition $\mathbf{u}_0 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$.

5. (15pts) Section 6.4, Problem 16

   Even if *A* is rectangular, the block matrix $B = \begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix}$ is symmetric:

   $B\mathbf{x} = \lambda\mathbf{x}$ is $\begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix}\begin{bmatrix} \mathbf{y} \\ \mathbf{z} \end{bmatrix} = \lambda\begin{bmatrix} \mathbf{y} \\ \mathbf{z} \end{bmatrix}$ which is $\begin{matrix} A\mathbf{z} = \lambda\mathbf{y} \\ A^T\mathbf{y} = \lambda\mathbf{z}. \end{matrix}$

   (a) Show that $-\lambda$ is also an eigenvalue, with the eigenvector (**y**,-**z**).

   (b) Show that $A^TA\mathbf{z} = \lambda^2\mathbf{z}$, so that λ^2 is an eigenvalue of A^TA.

   (c) If *A*=*I* (2 by 2) find all four eigenvalues and eigenvectors of *B*.

6. (20pts) True or false, with a good reason:

   (a) *A* can't be similar to *A* + *I*.

(b) A can't be similar to $-A$ unless $A=0$.

(c) A symmetric matrix can't be similar to a nonsymmetric matrix.

(d) An invertible matrix can't be similar to a singular matrix.

(e) If A is similar to B, then A^2 is similar to B^2.

(f) If A^2 is similar to B^2, then A is similar to B.

(g) $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ is similar to $\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$.

(h) $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ is similar to $\begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$.