Linear Algebra

Problem Set 2 **Spring 2012**

Due Tuesday, 13 March 2012 at 10:00 AM in EE208. This problem set covers Lecture 4-6. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (15pts) Find the inverses of the following matrices by inspection:

$$A=\begin{bmatrix}0&0&0&0&5\\1&0&0&0&0\\0&2&0&0&0\\0&0&3&0&0\\0&0&0&4&0\end{bmatrix} \text{ and } B=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&7&1&0&0\\0&0&0&1&2\\0&0&0&4&9\end{bmatrix}.$$

2. (20pts) Find the inverses (in any legal way) of

$$A=\begin{bmatrix}1&-1&1&-1\\0&1&-1&1\\0&0&1&-1\\0&0&0&1\end{bmatrix} \text{ and } B=\begin{bmatrix}1&2&3&4&5\\0&1&2&3&4\\0&0&1&2&3\\0&0&0&1&2\\0&0&0&0&1\end{bmatrix}.$$

3. (15pts) Use Gauss-Jordan method to solve $AX=B$, where

$$A=\begin{bmatrix}1&2&0\\0&2&1\\1&1&3\end{bmatrix},\ B=\begin{bmatrix}5&2&7\\3&5&9\\0&10&13\end{bmatrix}.$$

4. (15pts) If B is the inverse of A^2, show that AB is the inverse of A. Is it true that $AB=BA$? Why or why not?
5. (15pts) Suppose that A is a square matrix satisfying $A^3+2A^2+3A+I=0$. Find the inverses of A and $A+I$.
6. (20pts) Prove the following identities. Assume that all the inverses are legal.
   (a) $(A^{-1}+I)^{-1}=(A+I)^{-1}A=A(A+I)^{-1}$
   (b) $(A^{-1}+B^{-1})^{-1}=B(A+B)^{-1}A$