Linear Algebra

Problem Set 2 **Spring 2013**

Due Tuesday, 12 March 2013 at 12:00 PM in EE208. This problem set covers Lecture 5-7. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (10pts) If A is an n by n matrix, then the trace of A, denoted by tr(A), is defined to be the sum of diagonal entries of A. For example, if $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, then tr($A$)=1+5+9=15. Prove that if A is an m by n matrix and B is an n by m matrix, then tr(AB)=tr(BA).
2. (15pts) Find the inverse (in any legal way) of
$$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 7 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 3 & 5 \end{bmatrix}.$$
3. (20pts) Assuming that the stated inverses exist, prove the following equalities:
   (a) $(A^{-1} + B^{-1})^{-1} = A(A+B)^{-1}B$
   (b) $(I + AB)^{-1}A = A(I + BA)^{-1}$
4. (15pts) If $A = (I+B)^{-1}(I-B)$ and $B = \begin{bmatrix} 3 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$, find $(I+A)^{-1}$. (Please don't do a lot of calculations! Try to derive the formula of $(I+A)^{-1}$.)
5. (15pts) In each part solve the matrix equation for X.
   (a) $X \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ -3 & 1 & 5 \end{bmatrix}$
   (b) $X \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -1 & 0 \\ 6 & -3 & 7 \end{bmatrix}$
6. (15pts) Suppose $A = \begin{bmatrix} 1 & 0 & 0 & 2 & 3 \\ 0 & 1 & 0 & a & 1 \\ 0 & 0 & 1 & 0 & b \end{bmatrix}$ is row equivalent to

$$B = \begin{bmatrix} 2 & 6 & 3 & -2 & d \\ 1 & 4 & 3 & c & 19 \\ 0 & -1 & -1 & 1 & e \end{bmatrix},$$ i.e., there exists an invertible matrix E such that

$EA=B$. Find a, b, c, d, and e.

7. (10pts) Find the LU factorization of $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}$.