Linear Algebra

Problem Set 2 **Spring 2015**

Due Thursday, 19 March 2015 at 4:20 PM in EE106. This problem set covers Lecture 5-8. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (10pts) Prove that A is invertible if $a \neq 0$ and $a \neq b$ (find the pivots or A^{-1}):

$$A = \begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix}.$$

2. (15pts) True or false (with a counterexample if false and a reason if true):
   (a) A 3 by 3 matrix with a column of zeros is not invertible.
   (b) A 3 by 3 matrix with two identical rows is not invertible.
   (c) Every matrix with 1's down the main diagonal is invertible.
   (d) If A is invertible, then A^2 is invertible.
   (e) If A is invertible, then $A + A^T$ is invertible.
3. (10pts) Find the inverse (in any legal way) of

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 9 & 0 & 0 \\ 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}.$$

4. (20pts) Assuming that the stated inverses exist, prove the following equalities:
   (a) $A(A+B)^{-1}B = B(A+B)^{-1}A$
   (b) $(I + A^{-1})^{-1} = A(A+I)^{-1} = (A+I)^{-1}A$
5. (15pts) If $A = (I - B)(I + B)^{-1}$ and $B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 0 & 4 & 5 & 0 \\ 0 & 0 & 6 & 7 \end{bmatrix}$, find $(I + A)^{-1}$. (Please don't do a lot of calculations! Try to derive the formula of $(I + A)^{-1}$.)
6. (10pts) Given a 3 by 3 matrix A and four vectors

$$\mathbf{a} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}, \mathbf{d} = \begin{bmatrix} -2 \\ 2 \\ -3 \end{bmatrix} \text{ satisfying } A\mathbf{a} = \mathbf{b}, A\mathbf{b} = \mathbf{c}, A\mathbf{c} = \mathbf{d}, \text{ find } A\mathbf{d}.$$

7. (10pts) Suppose $A=\begin{bmatrix}1 & 2 & 3 & 1 & b\\ 2 & 5 & 3 & a & 0\\ 1 & 0 & 8 & 6 & c\end{bmatrix}$ is row equivalent to

$B=\begin{bmatrix}1 & 0 & 0 & -2 & 0\\ 0 & 1 & 0 & d & -1\\ 0 & 0 & 1 & 1 & e\end{bmatrix}$, i.e., there exists an invertible matrix E such that $EA=B$.

Find a, b, c, d, and e.

8. (10pts) Find the LU factorization of $A=\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & 2 & 3 & 4\\ 1 & 3 & 6 & 10\\ 1 & 4 & 10 & 20\end{bmatrix}$.