Linear Algebra

Problem Set 3 **Spring 2012**

Due Tuesday, 20 March 2012 at 10:00 AM in EE208. This problem set covers Lecture 7-9. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (15pts) Find the inverse of the following $n \times n$ matrix:

$$A = \begin{bmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & 0 & 1 & \cdots & 1 \\ 1 & 1 & 0 & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 0 \end{bmatrix}.$$

   Hint: Split A into $E - I$, where every entry of E is 1. Then try multiply some matrix on the right to get the identity matrix.

2. (15pts) Let

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 6 & 6 & 8 \\ 1 & 3 & 4 & 6 \\ 2 & 5 & 7 & 9 \end{bmatrix}.$$

   Find the permutation matrix P as well as LU factors such that $PA=LU$.

3. (20pts)
   (a) Split A in (2) into $B+C$, where $B = B^T$ is symmetric and $C = -C^T$ is anti-symmetric.
   (b) Find formulas for B and C involving A and A^T. We want $A = B + C$ with $B = B^T$ and $C = -C^T$.

4. (20pts) True or false. If true, give your reasoning; otherwise, give a counterexample.
   (a) If $A \neq 0$, it is impossible that $A^2 = 0$. Note that $A^2 = AA$.
   (b) If $A \neq 0$, it is impossible that $A^T A = 0$.
   (c) If A is symmetric then $A^k = AA \cdots A$ is symmetric, for every positive integer k.
   (d) If A and B are symmetric then $(A+B)(A-B)$ is symmetric.
   (e) If A and B are symmetric then $A^2 - B^2$ is symmetric.

5. (15pts) If you take powers of a permutation matrix, why is some P^k eventually equal to I? Find a 5 by 5 permutation matrix P so that $P^k \neq I$ for $k = 1, 2, 3, 4, 5$, and $P^6 = I$.

6. (15pts) Let A be an m by n matrix. Define $S = \begin{bmatrix} I_m & A \\ A^T & 0 \end{bmatrix}$. Use block elimination to find a block factorization $S = LDL^T$. Note that L has the form $L = \begin{bmatrix} I_m & 0 \\ X & I_n \end{bmatrix}$ and D has the form $D = \begin{bmatrix} Y & 0 \\ 0 & Z \end{bmatrix}$, where Y and Z are square matrices. Identify the condition that S is invertible and write down S^{-1} in the 2 by 2 block matrix form.