Linear Algebra

Problem Set 3 **Spring 2013**

Due Thursday, 21 March 2013 at 4:20 PM in EE208. This problem set covers Lecture 8-10. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (15pts) True or false. If true, give your reasoning; otherwise, give a counterexample.
   (a) If A is symmetric, then $A^k = AA\ldots A$ is symmetric, k is any positive integer.
   (b) If A and B are symmetric, then $\begin{bmatrix} B & A \\ A & B \end{bmatrix}$ is symmetric.
   (c) If A and B are symmetric, then ABA is symmetric.
   (d) If A and B are symmetric, then $ABAB$ is symmetric.
   (e) If A and B are symmetric, then $(A+B)^2$ is symmetric.
2. (15pts) Find the LDL^T factorization of the block matrix $B = \begin{bmatrix} I_n & A \\ A^T & 0 \end{bmatrix}$, where A is n by n. Note that L represents a block lower triangular matrix and D represents a block diagonal matrix. Prove that B is invertible if and only if A is invertible.
3. (10pts) Show that every n by n matrix A can be decomposed as $A=B+C$, where $B = B^T$ and $C = -C^T$. Note that C is called skew-symmetric.
4. (20pts)
   (a) Find a 3 by 3 permutation matrix with $P^3 = I$ (but not $P=I$).
   (b) Find a 5 by 5 permutation matrix so that the smallest power to equal I is P^6. That is, $P^k \neq I$, for $k = 1, 2, \ldots, 5$.
5. (15pts) Section 3.1, Problem 10
   Which of the following subsets of $\mathbb{R}^3$ are actually subspaces?
   (a) The plane of vectors (b_1, b_2, b_3) with $b_1 = b_2$.
   (b) The plane of vectors with $b_1 = 1$.
   (c) The vectors with $b_1 b_2 b_3 = 0$.
   (d) All linear combinations of $\mathbf{v} = (1,4,0)$ and $\mathbf{w} = (2,2,2)$.
   (e) All vectors that satisfy $b_1 + b_2 + b_3 = 0$.
   (f) All vectors with $b_1 \le b_2 \le b_3$.
6. (15pts) Section 3.1, Problem 18
   Let vector space $\mathbf{M}$ be the set of all 2 by 2 real matrices. True or false (check addition in each case by an example):
   (a) The symmetric matrices in $\mathbf{M}$ (with $A^T = A$) form a subspace.
   (b) The skew-symmetric matrices in $\mathbf{M}$ (with $A^T = -A$) form a subspace.

(c) The unsymmetric matrices in **M** (with $A^T \neq A$) form a subspace.

7. (10pts) Let A be a 2 by 2 symmetric matrix. Suppose every A can be written as a linear combination of B, C, and D. Give an example of B, C, and D.