Linear Algebra

Problem Set 3 **Spring 2015**

Due Thursday, 26 March 2015 at 4:20 PM in EE106. This problem set covers Lecture 9-12. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (15pts) Find the LDL^T factorization of the block matrix $\begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$, where A and C are n by n. Note that L represents a block lower triangular matrix and D represents a block diagonal matrix.
2. (10pts) Show that every n by n matrix A can be decomposed as $A=B+C$, where $B = B^T$ and $C = -C^T$. Express B and C in terms of A and A^T. Note that C is called skew-symmetric.
3. (15pts) Which of the following subsets of $\mathbb{R}^3$ are actually subspaces?
   (a) The plane of vectors (b_1, b_2, b_3) with $b_1 + b_2 = 0$.
   (b) The plane of vectors with $b_1 = 2$.
   (c) The vectors with $b_1 b_2 b_3 = 0$.
   (d) All linear combinations of $\mathbf{v} = (1,1,1)$ and $\mathbf{w} = (2,3,4)$.
   (e) All vectors that satisfy $b_1 + b_2 + b_3 = 1$.
   (f) All vectors with $b_1 \le b_2 \le b_3$.
4. (10pts) Let vector space **M** be the set of all 2 by 2 real matrices. True or false (check addition in each case by an example):
   (a) The symmetric matrices in **M** (with $A^T = A$) form a subspace.
   (b) The skew-symmetric matrices in **M** (with $A^T = -A$) form a subspace.
   (c) The unsymmetric matrices in **M** (with $A^T \ne A$) form a subspace.
   (d) The idempotent matrices in **M** (with $A^2 = A$) form a subspace.
5. (20pts) Let A be an n by n matrix. True or false (with a counterexample if false):
   (a) The vectors **b** that are not in the column space $C(A)$ form a subspace.
   (b) If $C(A)$ contains only the zero vector, then A is the zero matrix.
   (c) The column space of A^2 equals the column space of A.
   (d) The column space of $2A$ equals the column space of A.
   (e) The nullspace of $2A$ equals the nullspace of A.
   (f) A and A^T have the same nullspace.
   (g) A and A^T have the same column space.
   (h) A and A^T have the same free variables.
6. (10pts) Construct a 2 by 2 matrix whose nullspace equals its column space.
7. (10pts) Put as many 1's as possible in a 4 by 8 reduced row echelon matrix R so

that the free variables are 2, 4, 5, 6.

8. (10pts) Let

$$A = \begin{bmatrix} 2 & 4 & 6 & 2 \\ 2 & 5 & 7 & 3 \\ 2 & 3 & 5 & 1 \end{bmatrix}.$$

Find the nullspace matrix N of A and write down the complete solutions of $A\mathbf{x}=\mathbf{0}$.