Linear Algebra

Problem Set 3 **Spring 2016**

Due Tuesday, 22 March 2016 at 12:00 PM in EE105. This problem set covers Lectures 9-12. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your work.

1. (10pts) Consider the partitioned matrices

$$A = \begin{bmatrix} a & \mathbf{b}^T \\ 0 & C \end{bmatrix} \text{ and } S = \begin{bmatrix} 1 & 0 \\ 0 & D \end{bmatrix},$$

   where C and D are n by n matrices (D is invertible), a is a scalar, and $\mathbf{b}$ is a column vector with n components. Compute $S^{-1}AS$.

2. (10pts) Consider the partitioned matrix

$$A = \begin{bmatrix} I_n & \mathbf{b} \\ \mathbf{c}^T & 1 \end{bmatrix},$$

   where $\mathbf{b}$ and $\mathbf{c}$ are column vectors in $\mathbb{R}^n$. For which choices of $\mathbf{b}$ and $\mathbf{c}$ is A invertible? In these cases, what is A^{-1}?

3. (10pts) Which of the following sets are subspaces of $\mathbb{R}^3$?

   (a) $S = \{(x, y, z) \mid x + y + z = 1\}$

   (b) $S = \{(x, y, z) \mid xyz = 0\}$

   (c) $S = \{(x, y, z) \mid x \le y \le z\}$

   (d) $S = \{(x + y, y + z, z + x) \mid x, y, z \in \mathbb{R}\}$

   (e) $S = \{(x, y, z) \mid x = y = z\}$

4. (10pts) Consider the vectors $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m$ in $\mathbb{R}^n$. Prove that $\operatorname{span}\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m\}$ is a subspace of $\mathbb{R}^n$.

5. (15pts) In this problem, let

$$\mathbf{x} = \begin{bmatrix} 5 \\ 3 \\ -8 \end{bmatrix} \text{ and } \mathbf{y} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}.$$

   (a) Find a diagonal matrix A such that $A\mathbf{x} = \mathbf{y}$.

(b) Find a matrix A of rank 1, i.e., rank(A)=1, such that $A\mathbf{x} = \mathbf{y}$.

(c) Find an upper triangular matrix A such that $A\mathbf{x} = \mathbf{y}$. Also, it is required that all the entries of A on and above the diagonal be nonzero.

(d) Find a matrix A with all nonzero entries such that $A\mathbf{x} = \mathbf{y}$.

6. (15pts) Answer the following questions.

(a) Give an example of a 2 by 2 matrix A such that $C(A) = N(A) = \text{span}\left\{\begin{bmatrix}1\\3\end{bmatrix}\right\}$.

(b) Give an example of a matrix A such that $C(A)$ is plane with normal vector

$\begin{bmatrix}1\\2\\3\end{bmatrix}$ in $\mathbb{R}^3$.

(c) Give an example of a matrix A whose $N(A)$ is the plane $x + 2y + 3z = 0$ in $\mathbb{R}^3$.

(d) Give an example of a matrix A whose $N(A)$ is the line spanned by

$\begin{bmatrix}1\\-1\\2\end{bmatrix}$ in $\mathbb{R}^3$.

7. (10pts) Let

$$A = \begin{bmatrix}1 & 1 & 1 & 6\\1 & 2 & 3 & 4\\1 & 3 & 5 & 2\\1 & 4 & 7 & 0\end{bmatrix}.$$

Find a matrix B so that $N(B) = C(A)$.

8. (20pts) Answer the following questions.

(a) How many types of 3 by 2 matrices in reduced row-echelon form are there?

(b) How many types of 2 by 3 matrices in reduced row-echelon form are there?

(c) Consider an m by n matrix A. Can you transform rref(A) (reduced row echelon form of A) into A by a sequence of elementary row operations? Explain.

(d) If the rank of a 4 by 4 matrix A is 4, what is rref(A)?

(e) If the rank of a 5 by 3 matrix A is 3, what is rref(A)?