Linear Algebra

Problem Set 4 **Spring 2012**

Due Thursday, 29 March 2012 at 4:30 PM in EE208. This problem set covers Lecture 10-12. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (20pts) Which of the following are subspaces of $\mathbb{R}^4$?
   (a) All vectors that satisfies $x_1 - x_3 + x_4 = 0$.
   (b) All vectors that satisfies $x_1 + x_2 + x_3 + x_4 = 1$.
   (c) All vectors with $x_1 x_3 = 0$.
   (d) All vectors with $x_1 \le x_2$.
   (e) $\left\{(t, 1-t, 2t, 0) \middle| t \in \mathbb{R}\right\}$
   (f) $\left\{(t, -s+t, 2t, 4s) \middle| s, t \in \mathbb{R}\right\}$

2. (15pts) Let $S = \text{span}\{(1,1,1)\}$ and $T = \text{span}\{(1,2,3)\}$. True or false.
   (a) $S + T = \left\{\mathbf{x} + \mathbf{y} \middle| \mathbf{x} \in S, \mathbf{y} \in T\right\}$ is a subspace.
   (b) $S \cup T$ is a subspace.
   (c) $S + T = \text{span}\{S \cup T\}$

3. (15pts) All the following matrices are square. True or false. If true, give your reasoning; otherwise, give a counterexample.
   (a) The nullspace of $2A$ equals the nullspace of A.
   (b) If $A^2 = 0$, then $C(A) = \{\mathbf{0}\}$.
   (c) The column space of A^2 equals the column space of A.
   (d) The nullspace of A^2 equals the nullspace of A.

4. (10pts) Section 3.2, Problem 35, 36
   (a) If A is 4 by 4 and invertible, describe all vectors in the nullspace of the 4 by 8 matrix $B = \begin{bmatrix} A & A \end{bmatrix}$.
   (b) How is the nullspace $N(C)$ related to the spaces $N(A)$ and $N(B)$, if $C = \begin{bmatrix} A \\ B \end{bmatrix}$?

5. (20pts) Let A be a 3 by 4 matrix.

(a) Find a system of equations $A\mathbf{x}=\mathbf{b}$ whose solutions are

$$\text{span}\left\{\begin{bmatrix}1\\0\\5\\2\end{bmatrix},\begin{bmatrix}6\\0\\1\\2\end{bmatrix}\right\}.$$

(b) What is the reduced row echelon form R of A?

(c) Find the nullspace matrix N associated with R so that $C(N)=N(R)$.

(d) Prove that $C(N)=\text{span}\left\{\begin{bmatrix}1\\0\\5\\2\end{bmatrix},\begin{bmatrix}6\\0\\1\\2\end{bmatrix}\right\}$. Let S and T be two subspaces. To prove $S=T$, you can show that for every $\mathbf{x}\in S$, it follows that $\mathbf{x}\in T$, and thus $S\subseteq T$. Then prove $T\subseteq S$, and therefore $S=T$.

6. (20pts) Let $R=\begin{bmatrix}I_r & F\\0 & 0\end{bmatrix}$ be an m by n matrix and rank$R=r$.

(a) If $r=m$, find a B such that $RB=I$.

(b) If $r=n$, find a C such that $CR=I$.

(c) What is the reduced row echelon form of R^T? Write down in block matrix form.

(d) What is the reduced row echelon form of R^TR?

(e) Can you tell the reduced row echelon form of RR^T? If not, explain why not.