Linear Algebra

Problem Set 6 **Spring 2012**

Due Thursday, 26 April 2012 at 4:30 PM in EE208. This problem set covers Lecture 18-21. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (20pts) Find the range and kernel (like the column space and nullspace) of *T*:

   (a) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ -x_1 \end{bmatrix}$ (b) $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ x_1 + x_2 + x_3 \end{bmatrix}$

   (c) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (d) $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}$

   Identify which transformations are one-to-one and which are onto.
2. (15pts) Suppose $T : V \to W$ is a linear transformation. Let $n = \dim V$ and $m = \dim W$.
   (a) If *T* maps *V* onto *W*, what is the relationship between *m* and *n*?
   (b) If *T* is one-to-one, what can you say about *m* and *n*?
   (c) If *T* is one-to-one and maps onto *W*, what can you say about *m* and *n*?
3. (15pts) Let $T : V \to W$ be a linear transformation.
   (a) If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly dependent set in *V*, explain why the set $\{T(\mathbf{v}_1), T(\mathbf{v}_2), T(\mathbf{v}_3)\}$ is linearly dependent.
   (b) Suppose $\{\mathbf{u}, \mathbf{v}\}$ is a linearly independent set in *V*, but $\{T(\mathbf{u}), T(\mathbf{v})\}$ is a linearly dependent set. Show that there exists a nonzero vector **x** such that $T(\mathbf{x}) = \mathbf{0}$.
4. (15pts)
   (a) The set $B = \{1+t, 1+t^2, t+t^2\}$ is a basis for $\mathbf{P}^2$. Find the coordinate vector of $p(t) = 6 + 3t - t^2$.
   (b) Use coordinate vectors (with respect to any legal basis) to test the linear independence of the following polynomials: $1+2t^3$, $2+t-3t^2$, $-t+2t^2-t^3$.
5. (15pts)
   Suppose $T(\mathbf{v}_1) = \mathbf{w}_1$ and $T(\mathbf{v}_2) = \mathbf{w}_1 + \mathbf{w}_2$ and $T(\mathbf{v}_3) = \mathbf{w}_1 + \mathbf{w}_2 + \mathbf{w}_3$. Find the matrix *A* for *T* using these basis vectors. What input vector **v** gives $T(\mathbf{v}) = \mathbf{w}_3$?
6. (20pts) Suppose *T* is a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ and

$$T\left(\begin{bmatrix}1\\0\\1\end{bmatrix}\right)=\begin{bmatrix}1\\0\\0\end{bmatrix},\quad T\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right)=\begin{bmatrix}1\\1\\0\end{bmatrix},\quad T\left(\begin{bmatrix}0\\1\\1\end{bmatrix}\right)=\begin{bmatrix}1\\2\\-1\end{bmatrix}.$$

(a) Find the matrix representation of T with respect to the standard basis.

(b) Find the matrix representation of T with respect to the basis

$$\left\{\begin{bmatrix}1\\0\\1\end{bmatrix},\begin{bmatrix}1\\1\\0\end{bmatrix},\begin{bmatrix}0\\1\\1\end{bmatrix}\right\}.$$

(c) Find the matrix representation of T with respect to the basis

$$\left\{\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}2\\2\\3\end{bmatrix},\begin{bmatrix}0\\1\\1\end{bmatrix}\right\}.$$