Linear Algebra

Problem Set 8 **Spring 2012**

Due Thursday, 10 May 2012 at 4:30 PM in EE208. This problem set covers Lecture 25-27. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (20pts)
   Find the equation $y = \beta_0 + \beta_1 t$ of the least-squares line that best fits the data points $(2,1), (5,2), (7,3),$ and $(8,3)$. For the closest parabola $y = \beta_0 + \beta_1 t + \beta_2 t^2$ to the same four points, write down the unsolvable equations $A\mathbf{x} = \mathbf{b}$ in three unknowns $\mathbf{x} = (\beta_0, \beta_1, \beta_2)$. Set up the three normal equations $A^T A\hat{\mathbf{x}} = A^T\mathbf{b}$ (solution not required).
2. (20pts) Express the Gram-Schmidt orthogonalization of
   $$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \text{ as } A = QR.$$
3. (20pts) Let $A = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 \end{bmatrix}$. Find the projection matrices onto the column space, row space, nullspace, and left nullspace of A, respectively.
4. (15pts)
   (a) Find a basis for the subspace S in $\mathbb{R}^4$ spanned by all solutions of
   $$x_1 - x_2 + x_3 = 0.$$
   (b) Find a basis for the orthogonal complement $S^{\perp}$.
   (c) Find $\mathbf{b}_1$ in S and $\mathbf{b}_2$ in $S^{\perp}$ so that $\mathbf{b}_1 + \mathbf{b}_2 = \mathbf{b} = (1,1,-1,-1)$.
5. (10pts) Section 4.4, Problem 34
   $Q = I - 2\mathbf{u}\mathbf{u}^T$ is a reflection matrix when $\mathbf{u}^T\mathbf{u} = 1$. Two reflections give $Q^2 = I$.
   (a) Show that $Q\mathbf{u} = -\mathbf{u}$. The mirror is perpendicular to $\mathbf{u}$.
   (b) Find $Q\mathbf{v}$ when $\mathbf{u}^T\mathbf{v} = 0$. The mirror contains $\mathbf{v}$. It reflects to itself.
6. (15pts) Section 4.4, Problem 36

If A is m by n matrix with rank r, and $A = Q\begin{bmatrix} R \\ 0 \end{bmatrix} = \begin{bmatrix} Q_1 & Q_2 \end{bmatrix}\begin{bmatrix} R \\ 0 \end{bmatrix} = Q_1 R,$ where Q is an m by m orthogonal matrix, and R is an n by n upper triangular matrix. The n columns of Q_1 are an orthonormal basis for which fundamental subspace of A? The m-n columns of Q_2 are an orthonormal basis for which fundamental subspace of A?