Linear Algebra
Problem Set 8 **Spring 2013**

Due Tuesday, 28 May 2013 at 12:00 PM in EE208. This problem set covers Lecture 32-34. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (15pts) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$. Find the eigenvalues and corresponding eigenvectors of A. Is A diagonalizable? Why or why not.
2. (20pts) A 3 by 3 matrix A is known to have eigenvalues of $1, 1, 2$. Use this information to find the values of the following items (give the answers where possible):
   (a) The rank of A
   (b) The eigenvalues of A^T
   (c) The eigenvalues of A^3
   (d) The eigenvalues of $(A^2 + I)^{-1}$
   (e) The determinant of A^TA
   (f) The determinant of $A+2I$
   (g) The determinant of $A^{-1} + A$
   (h) The trace of $A+2I$
   (i) The trace of $3A+4A^T$
   (j) The trace of $P^{-1}AP$
3. (15pts) Find the eigenvalues of the following matrices.
   (a) $A = \begin{bmatrix} 5 & 5 & 0 & 7 \\ 0 & 1 & -3 & 3 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 2 \end{bmatrix}$
   (b) $B = \begin{bmatrix} 4 & 0 & 0 & 0 \\ -2 & 5 & 0 & 0 \\ 1 & 6 & 3 & 3 \\ 2 & 9 & -4 & -5 \end{bmatrix}$
   (c) $C = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

4. (20pts) True or false. Justify your answer.
   (a) If A is invertible, then 0 is not an eigenvalue of A.
   (b) If $A^2=0$, then the only eigenvalue of A is 0.
   (c) A nonzero vector cannot be the eigenvector corresponding to two different eigenvalues of A.
   (d) The sum of two eigenvectors of a matrix A is also an eigenvector.
   (e) Each eigenvector of A is also an eigenvector of A^T.
   (f) If A has distinct eigenvalues, then A is diagonalizable.
   (g) If A is row equivalent to I, then A is diagonalizable.
   (h) If A has eigenvalues 1, 1, and 1, then A is not diagonalizable.
   (i) If A is diagonalizable, then A^T is diagonalizable.
   (j) If A is diagonalizable, then A has linearly independent columns.
5. (15pts)
   (a) Compute A^8, where $A=\begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$.
   (b) Let $A=\begin{bmatrix} -3 & 12 \\ -2 & 7 \end{bmatrix}$. If $\mathbf{x}_1=\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\mathbf{x}_2=\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ are eigenvectors of A, use this information to diagonalize A. That is, write down $A=SDS^{-1}$, where D is a diagonal matrix.
6. (15pts)
   (a) Substitute $A=S\Lambda S^{-1}$ into the product $(A-\lambda_1 I)(A-\lambda_2 I)\cdots(A-\lambda_n I)$ and explain why this produces the zero matrix. We are substituting the matrix A for the number λ in the polynomial $p(\lambda)=\det(A-\lambda I)$. The Cayley-Hamilton theorem says that this product is always $p(A)=$zero matrix, even if A is not diagonalizable.
   (b) For a 3 by 3 matrix A, the Cayley-Hamilton theorem gives a matrix equation of the form $A^3+a_2A^2+a_1A+a_0I=0$. From this equation, you can get an expression of A^{-1}. Use the Cayley-Hamilton theorem to compute A^{-1}, where $A=\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.