Linear Algebra

Problem Set 9 **Spring 2012**

Due Thursday, 17 May 2012 at 4:30 PM in EE208. This problem set covers Lecture 28-30. Free feel to work with others, but the final write-up should be entirely based on your own understanding. Be sure to print your name and student ID on your homework.

1. (15pts) Section 5.1, Problem 3

   True or false, with a reason if true or counterexample if false:

   (a) The determinant of $I+A$ is $1+\det A$.

   (b) The determinant of ABC is $|A||B||C|$.

   (c) The determinant of $4A$ is $4|A|$.

   (d) The determinant of $AB-BA$ is zero. Try an example with $A=\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$.

2. (15pts) Section 5.1, Problem 17

   A skew-symmetric matrix has $K^T=-K$. Show that $\begin{vmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{vmatrix}=0$. Write down a 4 by 4 example with $|K|=1$.

3. (15pts) Section 5.2, Problem 13

   The n by n determinant C_n has 1's above and below the main diagonal:

   $$C_1=|0| \quad C_2=\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix} \quad C_3=\begin{vmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{vmatrix} \quad C_3=\begin{vmatrix}0 & 1 & 0 & 0\\ 1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\\ 0 & 0 & 1 & 0\end{vmatrix}.$$

   (a) What are these determinants C_1, C_2, C_3, C_4?

   (b) By cofactors find the relation between C_n and C_{n-1} and C_{n-2}. Find C_{10}.

4. (15pts) Section 5.2, Problem 34

   This problem shows in two ways that $\det A=0$ (the x's are any numbers and they may not be the same):

$$A = \begin{bmatrix} x & x & x & x & x \\ x & x & x & x & x \\ 0 & 0 & 0 & x & x \\ 0 & 0 & 0 & x & x \\ 0 & 0 & 0 & x & x \end{bmatrix}.$$

(a) How do you know that the rows are linearly dependent?

(b) Explain why all 120 terms are zero in the big formula (permutation formula) for det A.

5. (20pts) Section 5.2, Problem 25

Block elimination subtracts CA^{-1} times the first row $\begin{bmatrix} A & B \end{bmatrix}$ from the second row $\begin{bmatrix} C & D \end{bmatrix}$. This leaves the *Schur complement* $D - CA^{-1}A$ in the corner:

$$\begin{bmatrix} I & 0 \\ -CA^{-1} & I \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & D - CA^{-1}B \end{bmatrix}.$$

Take determinants of these block matrices to prove correct rules if A^{-1} exists:

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = |A| \cdot |D - CA^{-1}B| = |AD - CB| \text{ provided } AC = CA.$$

6. (20pts) Use determinants to answer the following questions.
   (a) A box has edges from (0,0,0) to (1,1,2) and (1,2,1) and (2,1,1). Find its volume.
   (b) Find the area of the parallelogram with sides from (0,0,0) to (1,1,1) and (2,0,3).
   (c) Find the area of the triangular with corners (2,1,1), (1,2,1) an (1,1,2).