Solutions to Problem Set 1 2012

1.

$$\begin{bmatrix} 4 & -1 & 3 & \vdots & b_1 \\ -2 & 2 & 0 & \vdots & b_2 \\ 1 & -2 & -1 & \vdots & b_3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & -1 & \vdots & b_3 \\ 0 & 7 & 7 & \vdots & b_1 - 4b_3 \\ 0 & -2 & -2 & \vdots & b_2 + 2b_3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & -1 & \vdots & b_3 \\ 0 & 1 & 1 & \vdots & \frac{b_1 - 4b_3}{7} \\ 0 & 1 & 1 & \vdots & \frac{b_2 + 2b_3}{-2} \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & -2 & -1 & \vdots & b_3 \\ 0 & 1 & 1 & \vdots & \frac{b_1 - 4b_3}{7} \\ 0 & 0 & 0 & \vdots & \frac{-7*(b_2 + 2b_3) - 2*(b_1 - 4b_3)}{2*7} \end{bmatrix}$$

∵pivots(A)＝2≠３ ∴$A\mathbf{x} = \mathbf{b}$ don't have a solution.

When $\frac{-7*(b_2 + 2b_3) - 2*(b_1 - 4b_3)}{2*7} = 0$, $2b_1 + 7b_2 + 6b_3 = 0$ $A\mathbf{x} = \mathbf{b}$ have infinite many solutions.

2.

$$\begin{bmatrix} 1 & a-1 & a-2 & \vdots & 2 \\ a & 2(a-1) & a-2 & \vdots & a+1 \\ a(a-2) & 0 & a & \vdots & -2a \end{bmatrix} \rightarrow \begin{bmatrix} 1 & a-1 & a-2 & \vdots & 2 \\ a-1 & a-1 & 0 & \vdots & a-1 \\ a(a-2) & 0 & a & \vdots & -2a \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} -(a-2) & 0 & a-2 & \vdots & -(a-3) \\ a-1 & a-1 & 0 & \vdots & a-1 \\ a(a-2) & 0 & a & \vdots & -2a \end{bmatrix} \rightarrow \begin{bmatrix} -(a-2) & 0 & a-2 & \vdots & -(a-3) \\ a-1 & a-1 & 0 & \vdots & a-1 \\ a(a-1)(a-2) & 0 & 0 & \vdots & -a(a-1) \end{bmatrix}$$

The last row is $\begin{bmatrix} a(a-1)(a-2) & 0 & 0 & \vdots & -a(a-1) \end{bmatrix}$

則 When a ＝０ or １ ,system has infinite many solutions.

當 a ＝２ , system has no solution.

當 a ≠０ , 1 and 2 , system has a unique solution.

3.

(1)

$$E_{21}=\begin{bmatrix}1&0&0&0\\1&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\to E_{21}A=\begin{bmatrix}1&-1&0&0\\0&1&-1&0\\0&-1&2&-1\\0&0&-1&2\end{bmatrix}$$

$$E_{32}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&1&1&0\\0&0&0&1\end{bmatrix}\to E_{32}E_{21}A=\begin{bmatrix}1&-1&0&0\\0&1&-1&0\\0&0&1&-1\\0&0&-1&2\end{bmatrix}$$

$$E_{43}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1&1\end{bmatrix}\to E_{43}E_{32}E_{21}A=\begin{bmatrix}1&-1&0&0\\0&1&-1&0\\0&0&1&-1\\0&0&0&1\end{bmatrix}$$

(2)

$$E_{43}E_{32}E_{21}=\begin{bmatrix}1&0&0&0\\1&1&0&0\\1&1&1&0\\1&1&1&1\end{bmatrix}$$

４.

$$E_4E_3E_2E_1 = P$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow E_2E_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \rightarrow E_3E_2E_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$E_4 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow E_4E_3E_2E_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

5.(1)用觀察法可知矩陣

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix} x = \begin{bmatrix} 1 & 0 & 4 & 4 & 5 \\ 2 & 2 & 4 & 5 & 5 \\ 3 & 0 & 4 & 6 & 6 \\ 4 & 0 & 4 & 7 & 7 \end{bmatrix}$$

令 $A = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 & 4 & 4 & 5 \\ 2 & 2 & 4 & 5 & 5 \\ 3 & 0 & 4 & 6 & 6 \\ 4 & 0 & 4 & 7 & 7 \end{bmatrix}$

矩陣Ｂ的第１行為Ａ矩陣第４行

矩陣Ｂ的第２行為Ａ矩陣第２行－第１行＊２

矩陣Ｂ的第３行為Ａ矩陣第３行＊４

矩陣Ｂ的第４行為Ａ矩陣第３行＊３＋第４行

矩陣Ｂ的第５行為Ａ矩陣第１行+第３行＊３＋第４行

故 $x=\begin{bmatrix}0 & -2 & 0 & 0 & 1\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 4 & 3 & 3\\ 1 & 0 & 0 & 1 & 1\end{bmatrix}$

$$x\begin{bmatrix}1 & 0 & 0 & 0\\ 1 & 1 & 0 & 0\\ 1 & 1 & 1 & 0\\ 1 & 1 & 1 & 1\end{bmatrix}=\begin{bmatrix}0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\end{bmatrix}$$

(2)

$$x\begin{bmatrix}1 & 0 & 0 & 0\\ 1 & 1 & 0 & 0\\ 1 & 1 & 1 & 0\\ 1 & 1 & 1 & 1\end{bmatrix}=\begin{bmatrix}0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\end{bmatrix}$$

令 A=$\begin{bmatrix}1 & 0 & 0 & 0\\ 1 & 1 & 0 & 0\\ 1 & 1 & 1 & 0\\ 1 & 1 & 1 & 1\end{bmatrix}$ B=$\begin{bmatrix}0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\end{bmatrix}$

矩陣B的第1列為A矩陣第4列-第3列
矩陣B的第2列為A矩陣第3列-第2列
矩陣B的第3列為A矩陣第2列-第1列
矩陣B的第4列為A矩陣第1列

故 $x=\begin{bmatrix}0 & 0 & -1 & 1\\ 0 & -1 & 1 & 0\\ -1 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\end{bmatrix}$

6.

$$\left[\begin{array}{ccc|ccc}3 & 2 & -2 & 1 & 0 & 0\\ -1 & -1 & 1 & 0 & 1 & 0\\ 1 & 3 & -2 & 0 & 0 & 1\end{array}\right]\to\left[\begin{array}{ccc|ccc}1 & 0 & 0 & 1 & 2 & 0\\ -1 & -1 & 1 & 0 & 1 & 0\\ 0 & 2 & -1 & 0 & 1 & 1\end{array}\right]\to\left[\begin{array}{ccc|ccc}1 & 0 & 0 & 1 & 2 & 0\\ 0 & -1 & 1 & 1 & 3 & 0\\ 0 & 2 & -1 & 0 & 1 & 1\end{array}\right]$$

$$\to\left[\begin{array}{ccc|ccc}1 & 0 & 0 & 1 & 2 & 0\\ 0 & -1 & 1 & 1 & 3 & 0\\ 0 & 0 & 1 & 2 & 7 & 1\end{array}\right]\to\left[\begin{array}{ccc|ccc}1 & 0 & 0 & 1 & 2 & 0\\ 0 & -1 & 0 & -1 & -4 & -1\\ 0 & 0 & 1 & 2 & 7 & 1\end{array}\right]\to\left[\begin{array}{ccc|ccc}1 & 0 & 0 & 1 & 2 & 0\\ 0 & 1 & 0 & 1 & 4 & 1\\ 0 & 0 & 1 & 2 & 7 & 1\end{array}\right]$$

$$X=\begin{bmatrix}1 & 2 & 0\\ 1 & 4 & 1\\ 2 & 7 & 1\end{bmatrix}$$