2013 spring HW1

1.

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & b_1 \\ 0 & 0 & 0 & b_2 \\ 2 & -4 & 6 & b_3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & b_1 \\ 0 & 0 & 0 & b_2 \\ 0 & 0 & 0 & b_3 - 2b_1 \end{array}\right]$$

$\Rightarrow b_2 \neq 0 \;\; or \;\; b_3 \neq 2b_1 \;\; no\, soultion$

$b_2 = 0 \;\; and \;\; b_3 = 2b_1 \;\;\; has\, soultion$

2.

(a)

$$\mathrm{E}_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b)

$$\mathrm{E}_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 7 & 1 \end{bmatrix}$$

(c)

$$p_{21} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad p_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$p_{32} * p_{21} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

3.

$$E_{21}\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \Rightarrow E_{32}\begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \Rightarrow E_{43}\begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & 1 & 0 \\ 0 & 0 & \frac{4}{3} & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & 1 & 0 \\ 0 & 0 & \frac{4}{3} & 1 \\ 0 & 0 & 0 & \frac{5}{4} \end{bmatrix}$$

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} E_{32} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} E_{43} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{3}{4} & 1 \end{bmatrix} U = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & 1 & 0 \\ 0 & 0 & \frac{4}{3} & 1 \\ 0 & 0 & 0 & \frac{5}{4} \end{bmatrix}$$

4.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 2 & -1 & 1 & 0 & 1 & 0 \\ 2 & -2 & 1 & 0 & 0 & 1 \end{array}\right] \overset{E_{21}^{2} E_{31}^{2}}{\Rightarrow} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{array}\right] \overset{E_{12}^{-1} E_{23}^{1}}{\Rightarrow} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{array}\right]$$

5.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \overset{E_{31}^{-1}}{\Rightarrow} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \overset{E_{13}^{1}}{\Rightarrow} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \overset{E_{21}^{1}}{\Rightarrow} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \overset{E_{12}^{-1}}{\Rightarrow} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \overset{E_{21}^{1}}{\Rightarrow} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \overset{E_{32}^{1}}{\Rightarrow} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

6.

(a) True

(b) False，$A=\begin{bmatrix}1&0&0&0\\0&2&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$ $B=\begin{bmatrix}1&0&0&0\\1&1&0&0\\0&0&1&0\\1&1&0&0\end{bmatrix}$ $AB=\begin{bmatrix}1&0&0&0\\2&2&0&0\\0&0&1&0\\1&1&0&0\end{bmatrix}$

(c) True

(d) False，$A=\begin{bmatrix}1&2\\0&1\end{bmatrix}$ $B=\begin{bmatrix}2&0\\1&1\end{bmatrix}$ $AB=\begin{bmatrix}4&2\\1&1\end{bmatrix}$ $A^2=\begin{bmatrix}1&4\\0&1\end{bmatrix}$ $B^2=\begin{bmatrix}4&0\\3&1\end{bmatrix}$

$$(AB)^2=(AB)(AB)=\begin{bmatrix}18&10\\5&3\end{bmatrix}\neq\begin{bmatrix}16&4\\3&1\end{bmatrix}=A^2B^2$$

(e)False，$A=\begin{bmatrix}1&2\\0&1\end{bmatrix}$ $B=\begin{bmatrix}2&0\\1&1\end{bmatrix}$ $A+B=\begin{bmatrix}3&2\\1&2\end{bmatrix}$

$$(A+B)^2=A^2+AB+BA+B^2=\begin{bmatrix}11&10\\5&6\end{bmatrix}\neq\begin{bmatrix}13&8\\5&4\end{bmatrix}=A^2+2AB+B^2$$

7.

(a) $X=\begin{bmatrix}1&0&0&1&0\\1&-1&0&1&-3\\1&1&0&1&0\\1&0&3&2&6\end{bmatrix}$

(b) $X=\begin{bmatrix}-1&1&-1&1\\1&-1&1&0\\-1&1&0&0\\1&0&0&0\end{bmatrix}$

8.

The same is true for all other columns of C.

$\because$ AC=I

$\therefore$ BAC = B(AC) = BI = B

$\because$ BA=I

$\therefore$ BAC = (BA)C = IC = C

$\Rightarrow$ BAC = B(AC) = (BA)C = BI = IC

$\Rightarrow$ left-inverse B equals the right-inverse C.