2013 spring HW3

1.

(a) True

$\because$ A is symmetric. $\therefore A=A^T$ $\because AA=AA^T=A^TA^T=(AA)^T$ $\therefore$ AA is still symmetric.

$\Rightarrow A^k=AA.......A=(A^k)^T$ is symmetric.

(b) True

$\because \begin{bmatrix} B & A \\ A & B \end{bmatrix}^T = \begin{bmatrix} B^T & A^T \\ A^T & B^T \end{bmatrix} = \begin{bmatrix} B & A \\ A & B \end{bmatrix}$ $\therefore \begin{bmatrix} B & A \\ A & B \end{bmatrix}$ is symmetric.

(c) True

$\because (ABA)^T=A^TB^TA^T = ABA$ $\therefore$ ABA is symmetric.

(d) False

Suppose $A=\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ $B=\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$ ，$ABAB=\begin{bmatrix} 19 & 35 \\ 14 & 26 \end{bmatrix}$ which is not symmetric.

(e) True

$\because$ A B are symmetric. $\therefore A=A^T$, $B=B^T$ $\therefore A+B=A^T+B^T=(A+B)^T$ A+B is symmetric.

$\Rightarrow ((A+B)^2)^T=((A+B)(A+B))^T=(A+B)^T(A+B)^T=(A+B)(A+B)=(A+B)^2$

$\Rightarrow (A+B)^2$ is symmetric

2.

$$B=\begin{bmatrix} I_n & A \\ A^T & 0 \end{bmatrix}=LU=\begin{bmatrix} I_n & 0 \\ A^T & I_n \end{bmatrix}\begin{bmatrix} I_n & A \\ 0 & -A^TA \end{bmatrix}=LDL^T=\begin{bmatrix} I_n & 0 \\ A^T & I_n \end{bmatrix}\begin{bmatrix} I_n & 0 \\ 0 & -A^TA \end{bmatrix}\begin{bmatrix} I_n & A \\ 0 & I_n \end{bmatrix}$$

B is invertible.

$\Leftrightarrow B^{-1}=(LDL^T)^{-1}=\begin{bmatrix} I_n & -A \\ 0 & I_n \end{bmatrix}\begin{bmatrix} I_n & 0 \\ 0 & (-A^TA)^{-1} \end{bmatrix}\begin{bmatrix} I_n & 0 \\ -A^T & I_n \end{bmatrix}$ exist.

$\Leftrightarrow$ $(-A^TA)^{-1}=-A^{-1}(A^{-1})^T$exist.

$\Leftrightarrow$ A^{-1}exist.

$\Leftrightarrow$ A is invertible.

3.

$$A=B+C \Rightarrow A^T=B^T+C^T=B-C \Rightarrow \begin{cases} B=\dfrac{A+A^T}{2} \\ C=\dfrac{A-A^T}{2} \end{cases} \Rightarrow \text{B C always exist.}$$

$\Rightarrow$ A can be decomposed as A=B+C .

4.

(a) $P=P_{32}P_{21}=\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$ ， $P^3=I$

(b) $P=\begin{bmatrix}\begin{matrix}P_2\end{matrix} & \begin{matrix}0&0&0\\0&0&0\end{matrix}\\ \begin{matrix}0&0\\0&0\\0&0\end{matrix} & P_3\end{bmatrix}$ which $P_2=\begin{bmatrix}0&1\\1&0\end{bmatrix}$ $P_3=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$ ， $P=\begin{bmatrix}0&1&0&0&0\\1&0&0&0&0\\0&0&0&1&0\\0&0&0&0&1\\0&0&1&0&0\end{bmatrix}$ ， $P^6=I$

5.

(a) Yes

(b) No， Let $V=\{(b_1,b_2,b_3) \mid b_1=1\}$, $(1,1,1)\in V$,but $2(1,1,1)=(2,2,2)\notin V$

(c) No， Let $V=\{(b_1,b_2,b_3) \mid b_1b_2b_3=0\}$, $(0,1,1)\in V$,$(1,0,0)\in V$

but $(0,1,1)+(1,0,0)=(1,1,1)\notin V$

(d) Yes

(e) Yes

(f) No， Let $V=\{(b_1,b_2,b_3) \mid b_1\le b_2\le b_3\}$, $(1,2,3)\in V$,but $-1(1,2,3)=(-1,-2,-3)\notin V$

6.

(a) Yes

1. A and B are symmetric $\in M(A^T=A)$

$\Rightarrow A+B=A^T+B^T=(A+B)^T$ is also symmetric $\in M$

2. $c\in R\Rightarrow cA=cA^T$ is symmetric $\in M$

(b) Yes

1. A and B are skew-symmetric $\in M(A^T=-A)$

$\Rightarrow (A+B)^T=A^T+B^T=-A-B=-(A+B)$ is also skew-symmetric $\in M$

2. $c\in R\Rightarrow cA^T=-cA$ is skew-symmetric $\in M$

(c) No

A and B are unsymmetric $\in M(A^T\neq A)$, but A+B might be a symmetric matrix.

$A=\begin{bmatrix}1&2\\1&0\end{bmatrix}\in M$ ， $B=\begin{bmatrix}1&0\\1&1\end{bmatrix}\in M$ ， $A+B=\begin{bmatrix}2&2\\2&1\end{bmatrix}\notin M$

7.

$A=\begin{bmatrix}a&b\\b&c\end{bmatrix}=aB+bC+cD=a\begin{bmatrix}1&0\\0&0\end{bmatrix}+b\begin{bmatrix}0&1\\1&0\end{bmatrix}+c\begin{bmatrix}0&0\\0&1\end{bmatrix}$