Solution to Problem Set 9

1.

(a) false

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad I + A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\det(A) = 1 \quad \neq \det(I+A) = 4$$

(b) true

$$\det(ABC) = \det(A)\det(BC) = \det(A)\det(B)\det(C)$$

(c) false

$$\det(4A_{n*n}) \quad = 4^n \det(A_{n*n})$$

(d)false

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \quad BA = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \qquad \text{AB-BA} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

2.

(a)

$$\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix} = -a * \begin{vmatrix} -a & c \\ -b & 0 \end{vmatrix} + b \begin{vmatrix} -a & 0 \\ -b & -c \end{vmatrix} = -abc + abc = 0$$

(b)

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

3.

(a)

$$C_1 = |0| = 0 \qquad C_2 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 \qquad C_3 = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = 0$$

$$C_4 = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix} = -1 * \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = 1$$

(b)

$$C_n = \begin{bmatrix} 0\ 1\ 0 \cdots\cdots 0 \\ 1\ 0\ 1 \cdots\cdots 0 \\ 0\ 1 & \\ \vdots\ \vdots & C_{n-2} \\ 0\ 0 & \end{bmatrix} = -1 \begin{bmatrix} 1\ 1 \cdots\cdots 0 \\ 0 & \\ \vdots & C_{n-2} \\ 0 & \end{bmatrix} = -C_{n-2}$$

$C_{10} = -C_8 = C_6 = -C_4 = -1$

4.

(a)

$$Let\ \ A = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \quad a_3, a_4, a_5 = \begin{bmatrix} 0 & 0 & 0 & x & x \end{bmatrix} \quad (x\ is\ any\ \ different\ \ numbers)$$

$Because\ \ c_1a_3 + c_2a_4\ \ (c_1, c_2 \neq 0)\ \ can\ \ combination\ \ a_5$

$\rightarrow the\ \ rows\ \ are\ \ linearly\ \ dependent$

(b)

$\det A = a_{11}a_{22}a_{33}a_{44}a_{55} - a_{12}a_{21}a_{33}a_{44}a_{55} \cdots\ \ (120\ \ terms)$

$\rightarrow every\ \ term\ \ must\ \ contains\ \ zero\,(according\ \ permetation\ \ formula)$

$\therefore \det A = 0$

5.

$Use\ \ the\ \ block$ determinants

$$\rightarrow \begin{vmatrix} I & 0 \\ -CA^{-1} & I \end{vmatrix} = |I| \times |I| = 1$$

$$\rightarrow \begin{vmatrix} I & 0 \\ -CA^{-1} & I \end{vmatrix} \begin{vmatrix} A & B \\ C & D \end{vmatrix} = \begin{vmatrix} A & B \\ 0 & D - CA^{-1}B \end{vmatrix} = \begin{vmatrix} A & B \\ C & D \end{vmatrix}$$

$$\therefore \begin{vmatrix} A & B \\ C & D \end{vmatrix} = |A| \times |D - CA^{-1}B| = |A \times (D - CA^{-1}B)| = |AD - ACA^{-1}B|,\ if\ AC = CA$$

$$\rightarrow \begin{vmatrix} A & B \\ C & D \end{vmatrix} = |AD - CB|$$

6.

(a)

$$\left\| \begin{matrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{matrix} \right\| = 4$$

(b)

$$|(1,1,1)\times(2\ ,0,3)|=|(3,-1,-2)|=\sqrt{14}$$

($\times$: cross)

(c)

$$(2-1,1-2,1-1)=(1,-1,0)$$
$$(2-1,1-1,1-2)=(1,0,-1)$$
$$1/2 * |(1,-1,0)\times(1,0,-1)|=|(1,1,1)|=\frac{\sqrt{3}}{2}$$